APPLIED STATICS

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APPLIED STATICS

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PREFACE TO FIFTH EDITION.

It is the intention in the following pages to provide an elementary text to be used in conjunction with the lectures on Statics as given in the Faculty of Applied Science and Engineering at Toronto University. Recognizing the fact that the student very often wastes a great deal of time in working out the steps that are omitted in most mathematical text-books, the writer has endeavored to give each step in full, sometimes at the expense of conciseness.

Chapters XIV., XV., XVI. and the Appendix have been added to the original edition. The reader will no doubt notice many repetitions in the problems presented in the first three named chapters. These problems originally appeared in serial form in "The Canadian Engineer," which, of course, necessitated a large amount of repetition from week to week. It was thought better, however, to publish the problems in this book just as they were, for, as explained in the previous paragraph, this book is for the beginner and not the advanced student, who knows how and where to look for previous reasoning.

The writer wishes to acknowledge his indebtedness to Professor C. H. C. Wright, upon whose course of lectures the following work is based, and to Professor W. J. Loudon for many valuable suggestions.

THOS. R. LOUDON.

Toronto, October 1st, 1912.

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APPLIED STATICS

CHAPTER I.

MECHANICS.

In its general interpretation, the subject of Mechanics treats of the Laws of Motion, of the relations that exist between matter and motion, and of the laws of equilibrium of what are termed **forces.**

Force, although popularly regarded as a definite, concrete idea, is the same sort of thing as velocity or acceleration, and is measured by the rate at which a mass changes its velocity.

In Engineering Mechanics, however, it may be considered as a definite quantity, capable of being measured, and represented by a number, the unit of which is the weight of one pound avoirdupois.

We speak, therefore, in popular phraseology of a force of ten pounds, or of a force **acting on a body**, or of forces **acting in certain directions**, just as if a force were a concrete thing, these being merely convenient, although scientifically misleading, modes of expression, which have been so ingrained into the whole subject of Mechanics that it would require a vast amount of circumlocution to avoid their use.

The following subdivision of Mechanics is generally used:—

1. Kinematics, which treats only of the geometry of motion.

2. Dynamics, which treats of the relations between matter and motion.

3. Statics, which deals with the laws of equilibrium of forces.

Although the subject of Statics alone will be considered here, yet the student should bear in mind that the second section, that of Dynamics, treats of Newton's Laws of Motion, which are given here because of their importance, especially the Third Law, the applications of which are continually occurring in statical problems.

Newton's Laws of Motion.

I. A body continues in a state of rest or uniform motion in a straight line till disturbed from such a state by an impressed force.

II. Change of motion is proportional to the impressed force, and takes place in the direction of the impressed force.

III. **Action and reaction are equal and opposite.**

CHAPTER II.

STATICS.

Taking the definition of the subject of Statics as given in the last chapter, it is seen that the consideration of motion does not enter into statical problems. It may be said, however, that although the analysis of motion produced or the tendency to produce motion does not constitute a part of the subject of Statics, it is often very convenient to examine the motion, or tendency to produce motion, in order that the properties of a given force or set of forces may be intelligently obtained.

The only forces considered in this treatise are coplanar forces, or forces acting in one plane.

It is advisable, before attempting the discussion of statical problems, that the student familiarize himself with the meanings of the terms given below:—

Magnitude—A property which admits of being measured. Quantities having this property may be relatively compared.

Direction—The common idea of direction is in reality compounded of two ideas, viz.: The line along which a certain manifestation may take place, or along which one body lies relative to another body; and which **way** along the line the manifestation takes place, or the one body lies relative to the other body. Direction, in the following pages, will be understood to mean merely the line along which action is manifested; concisely, **line of action.**

Sense—One of two **ways** in which a magnitude may be described or generated along a direction.

This distinction between Direction and Sense is more clearly seen by examples:—

A body is said to move vertically upward; i.e., the body moves in the vertical direction with an upward sense.

A building is said to lie east of another building; i.e., the first building lies in the direction of a parallel of latitude relative to the other building, and in an easterly sense.

A Vector.—A line conceived to have fixed length and direction in space, but whose position is not limited; i.e., the line may be drawn anywhere.

A Vector Quantity.—Any quantity having the properties, Magnitude, Direction, and Sense, is known as a vector quantity; e.g., Force, Velocity, Displacement, Acceleration. Such quantities may be graphically represented by Vectors.

A Rigid Body is one whose original form cannot be altered; i.e., the body cannot be compressed, extended, broken, or bent. Such a body, of course, exists merely in theory.

CHAPTER III.

GRAPHICAL REPRESENTATIONS—THE RESULTANT—COMPONENTS.

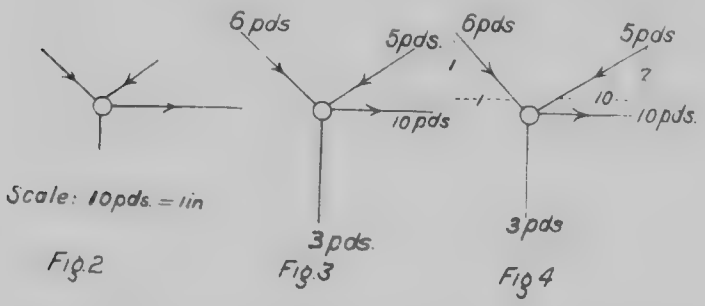
Graphical Representation of Force.

A Force, being a Vector Quantity, may be represented by a right line; for the direction of the line may represent the line of action of the force, the magnitude and sense being represented by first cutting off a portion of the line to some convenient scale of magnitude, and by then placing on this portion an arrow head pointed in the sense of action of the force.

Fig. 1.

Fig. 1 represents a force of ten pounds magnitude acting in the horizontal direction, with sense to the left, as indicated by the arrow head. The scale of magnitudes in this case is $\frac{1}{4}$ inch = 1 pound.

Fig. 2 represents a set of forces acting at a point. In this diagram the magnitudes of the forces are represented by the lengths of the various lines. It is often more convenient, however, merely to represent the directions and senses accurately, and to place the numerical values of the magnitudes beside the lines representing



the respective forces as in Fig. 3. Sometimes, when general conditions are merely wished in the form of a note, the directions as well as the magnitudes are only relatively represented, and the actual direction ratios, as well as the magnitudes, jotted down on the diagram as in Fig. 4

A diagram, such as either Fig. 2, Fig. 3, or Fig. 4, in which is given all the relative information of a set of forces, will be referred to as the **Statical Diagram** of the set of forces in question.

Bow's Notation.

In order that the graphical representation of any particular force may be intelligently and concisely referred to, a system of lettering known as Bow's Notation will be used throughout the work whenever possible. In this notation, the areas between the lines representing the forces are lettered, and a force is known by the letters to the areas on each side of the line representing that particular force.

Referring to the Statical Diagram (Fig. 5), the force of 50 pounds is known as either the force AB or BA, the force of 30 pounds as either the force BC or CB, and the force of 60 pounds as either the force CA or AC

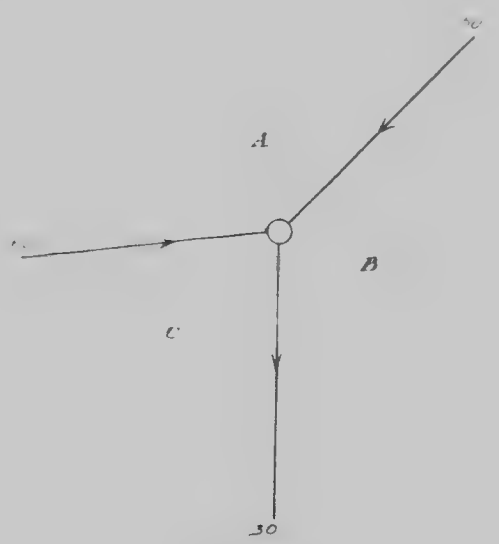


Fig. 5.

The Resultant Force is that one force which may replace and produce the same effect as a set of forces.

It is usual to refer to the Resultant Force merely as the **Resultant.**

It is evident that no resultant can be found for a set of forces in equilibrium. Because, if that could be done, there would be a single force (the resultant) acting on the body, and, therefore, motion would take place, which is impossible with equilibrium.

The Equilibriant or Balancing Force of a set of forces is that one force which, when acting with the set of forces, produces equilibrium.

The Equilibriant and Resultant of any given set of forces are of equal magnitude, and act in the same direction but with opposite senses; or concisely, they are two equal and opposite forces. For, since the Resultant produces the same effect as the set of forces, it is manifest from definition that the Equilibriant would, when acting with the Resultant, produce equilibrium. Experimentally, it may be shown that the only possible manner in which two forces acting on the same body can give a state of equilibrium is that they be of equal magnitude and act in the same line with opposite senses.

Graphical Determination of the Magnitude, Direction, and Sense of the Resultant of a Set of Forces.

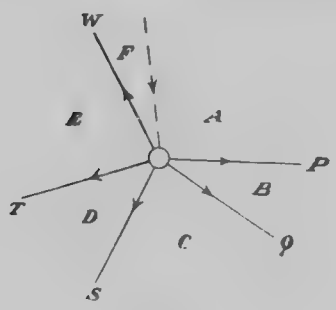


Fig. 6.

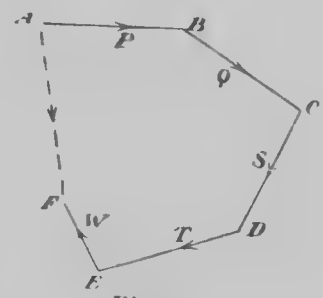


Fig. 6a.

Let there be given any set of forces, but, to make the problem clear, let us take a set of forces acting at a point. Fig. 6 is the Statical diagram of this set. Choose any initial point A as in Fig. 6a. From this point draw a line AB to represent **accurately** the force P in Fig. 6. From the point B, the line BC is drawn to represent the force Q (Fig. 6). In the same way the lines CD, DE, and EF are drawn to represent the forces S, T, and W, respectively. If now, the initial point A be joined to the final point F, then AF represents fully

the Resultant of the forces P, Q, S, T, and W. **The proof of this statement is experimental.**

The accuracy of the representation of the resultant in this last diagram depends entirely upon the care with which the relative directions and magnitudes of the forces are represented. If the directions of the forces relative to one another are represented accurately in the statical diagram, the simplest way of constructing the polygon is to draw the composing lines parallel to corresponding lines in the statical diagram.

The only restriction that is placed upon the construction of this polygon is that **the forces must be taken in such order that the sense marks in the polygon point continuously from the initial to the final point.** The sense mark of the resultant is always from the initial to the final point, and is, as it were, counter to the other sense marks.

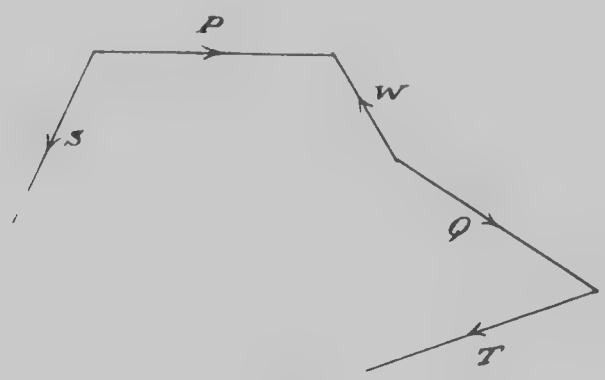


Fig. 7

Fig. 7 shows an improper way of constructing the diagram. It will be noticed that the sense marks do not point continuously around the diagram from the initial to the final point. The resultant may, however, be found by a different arrangement of the representations than that shown in Fig. 6a. Fig. 8 shows a different diagram properly constructed with sense marks pointing continuously from the initial to final point. The resultant as found in this last case will have exactly the same magnitude, direction and sense as that found in Fig. 6a.

In the case where the forces do not act at a point, the procedure is exactly the same, the forces being chosen in such order that the sense marks on their representations point continuously from the initial to the final point of the diagram.

Diagrams such as Fig. 6a and Fig. 8 will be referred to in future as **Vector Polygons** or **Vector Diagrams,** and must not be confused with the Statical Diagram. The distinction lies in the fact that the Statical Diagram gives all information regarding the actual relatve positions of the forces as they lie in space, as well as the properties of the forces; whereas, only the magnitudes, directions, and senses of the forces need be known to construct the Vector Polygon.

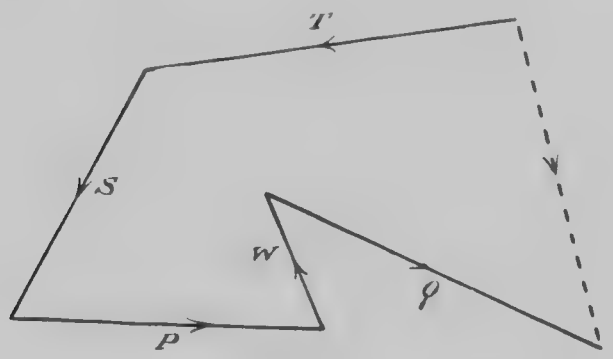


Fig. 8.

For a set of forces acting at a point, the resultant must act through this point in order to produce the same effect. Also, since the equilibriant acts in the same direction but in the opposite sense to the resultant, the equilibriant of a set of forces acting at a point must also act through the point. This gives a method of locating the resultant and equilibriant of a set of forces acting at a point, but for the general case, where the forces do not act at a point, the location of the resultant and equilibriant does not admit of such an easy solution, as will be seen later.

First Graphical Condition of Equilibrium.

It has been shown that a set of forces in equilibrium can have no resultant. Therefore, if the vector polygon be constructed for such a set of forces, the distance between the initial and final points, which represents the

resultant, must be equal to zero; i.e., the initial and final points will coincide, or, in other words, the polygon closes. From this, then, **the first graphical condition of equilibrium is that the vector polygon must close.**

Components.

It has been pointed out that a set of forces may be replaced by a single force. Conversely, a force may be replaced by a set of two or more forces. The forces comprising such a set are known as **Components** of the replaced force, and, since the force is interchangeable with any set of its components, it follows that **a force is the resultant of a set of its components.** This may be more clearly seen from Fig. 9.

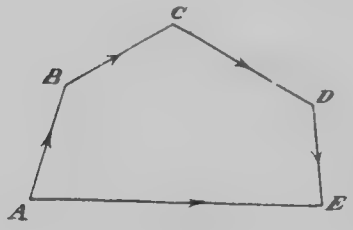


Fig. 9.

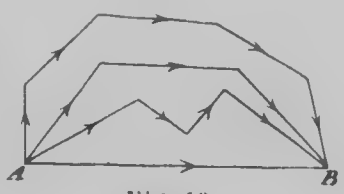


Fig. 10.

Let AE represent any force. AE is the resultant of AB, BC, CD, and DE; i.e., AE may replace AB, BC, CD, and DE. Conversely, the set AB, BC, CD, and DE may replace AE; i.e., they are a set of components. AE, the original force, is then the resultant of any of its component sets.

From this last statement it is seen that, if from the initial point of the representation of a force a number of lines be drawn continuously till the final point of the original representation is reached, these lines will represent a set of components. Fig. 10 shows a number of sets of components to the force AB. It is easily seen that an infinite number of such sets may be found.

To any force there may be found an infinite number of sets of components.

Resolved Parts.

In many of the problems of Statics, it is convenient to consider a force as replaced by a set of components, and in order to give definiteness to the problem, it is

usual to replace the original force by two components whose directions are inclined at right angles to one another. Such a pair of components are known as **Resolved Parts**, and since a pair of resolved parts must always act perpendicularly to one another, the direction

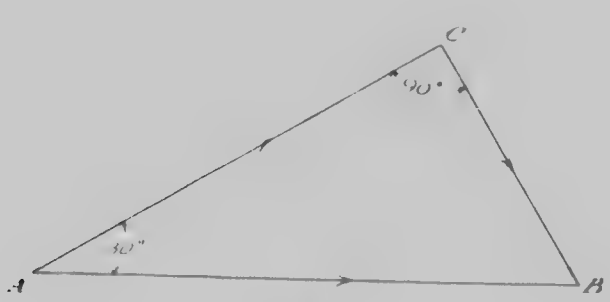


Fig. 11.

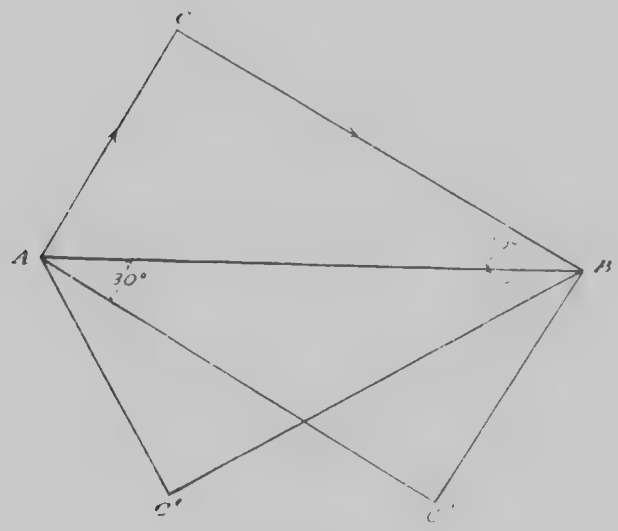


Fig. 12.

of one resolved part being given fixes the direction of the remaining resolved part. The operation of finding the value of a pair of resolved parts in any given directions that are perpendicular to one another, is known as **resolving** the original force in those directions.

Example:

A force of 50 pounds acts horizontally to the right. Find the magnitude of a pair of resolved parts, one of which is inclined to the horizontal at thirty degrees.

Until the student has become familiar with the operation of resolving forces, it is advisable to draw a rough graphical representation of the force and the required resolved parts, and then, from the data, find by trigonometrical means the required values.

Let AB (Fig. 11) represent the given force. AC and CB evidently represent the required resolved parts, for AC makes an angle of 30° with the horizontal, and CB is perpendicular to AC. Then

$$AC = AB \cos 30 = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3}.$$

$$CB = AB \sin 30 = 50 \times \frac{1}{2} = 25,$$

which gives the magnitudes of the resolved parts in terms of the original force.

There are three other correct solutions to the problem which are shown in Fig. 12.

AC, CB; AC′, C′B; AC″, C″B are resolved parts which fill the requirements of the problem.

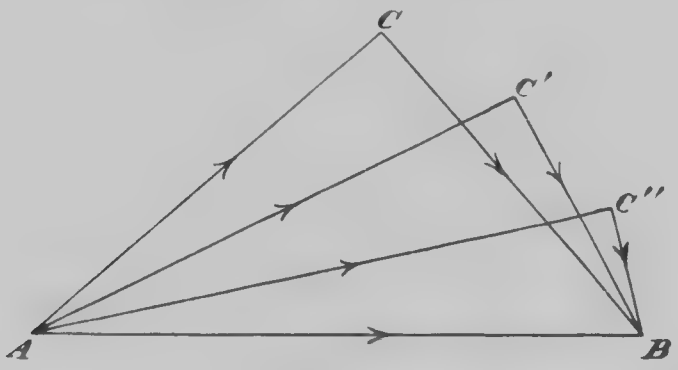


Fig. 13.

Resolved Parts Perpendicular and Parallel to a Given Force.

AC, CB, AC′, C′B, and AC″, C″B (Fig. 13) represent sets of resolved parts of AB. The directions of AC, AC′ and AC″ are respectively parallel and coincident with the direction of AB. The directions of CB, C′B,

and CB [illegible] nearly perpendicular to AB. Also, AC'' is nearer the value of AB than AC', and C''B is closer to the value zero than CB. Consequently, if AB be resolved in directions coincident with and perpendicular to itself, the magnitudes of the resolved parts would be respectively equal to AB and zero. In general, then, **the magnitude of a resolved part perpendicular to a given force is equal to zero, and the corresponding resolved part, whose direction must be coincident with the given force, is equal to the given force.**

Horizontal and Vertical Resolved Parts.

In engineering statical problems, it is customary, unless otherwise stated, to confine attention to resolved parts in the horizontal and vertical directions, and in order that these may be concisely referred to, the horizontal resolved part is called the X, and the vertical resolved part the Y of the force in question. Furthermore, as will be seen, since the solution of many problems depends upon the algebraic summation of the X's and Y's of a set of forces, a convention as to the positive and negative sign of X's and Y's must be agreed upon.

Horizontal resolved parts (X's) acting to the right and vertical resolved parts (Y's) acting upward shall be considered positive.

Magnitudes of the X and Y of any Force whose Inclination to the Horizontal is Given.

Let AB (Fig. 14) represent a force of magnitude P. If the inclination of the line of action of the force to the horizontal be the angle α as shown, then AC and CB represent X_P and Y_P respectively.

$$X_P = AC = AB \cos \alpha = P \cos \alpha.$$
$$Y_P = CB = AB \sin \alpha = P \sin \alpha.$$

If P act to the left as represented by AB', and the angle of inclination still remain α, X_P is negative, but its magnitude is still given by P cos α. The Y_P is positive and still of magnitude P sin α.

It is seen, then, that the magnitude of the X of any force, irrespective of its sign, is given by the **product of the magnitude of the force and the cosine of the**

angle of inclination to the horizontal. The magnitude of the Y of any force is given by the **product of the magnitude of the force and the sine of the angle of inclination to the horizontal.**

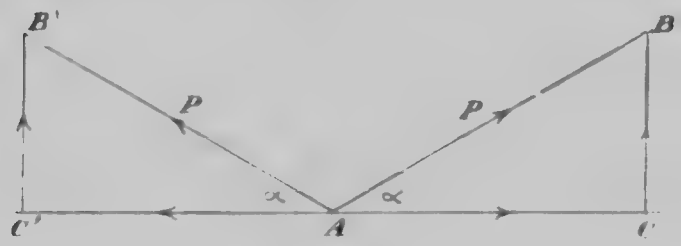


Fig. 14.

The X and Y of Horizontal and Vertical Forces.

If P be a horizontal force, then $\alpha = 0$, and

$$X_P = P \cos \alpha = P \cos 0 = P.$$
$$Y_P = P \sin \alpha = P \sin 0 = 0$$

If P be a vertical force, then $\alpha = 90$, and

$$X_P = P \cos \alpha = P \cos 90 = 0.$$
$$Y_P = P \sin \alpha = P \sin 90 = P.$$

That is: **The horizontal resolved part of a horizontal force and the vertical resolved part of a vertical force are both equal to the force itself.**

The vertical resolved part of a horizontal force and the horizontal resolved part of a vertical force are both equal to zero.

CHAPTER IV.

MOMENTS—ANALYTICAL CONDITIONS OF EQUILIBRIUM—COUPLES.

If a body be pinned at some one point and acted on by any force not passing through this point, there will be produced a rotation, or a tendency to rotate. If a new point be chosen, or if a new force be introduced, this tendency to rotate will also change.

It may be shown that the turning effect varies as the product of the magnitude of the force and the perpendicular distance of the force from the point. To this product is given the name of the **Moment** of the force.

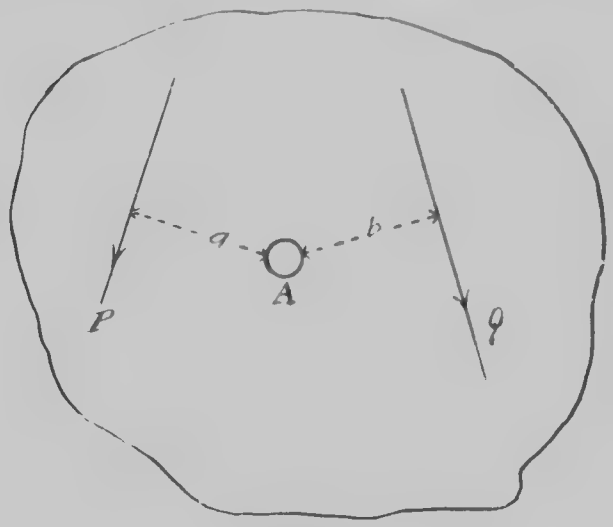


Fig. 15.

Definition:

The moment of a force about a given point is the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force.

Since it is often necessary to find the algebraic sum of the moments of a set of forces about some one point, a convention as to the positive and negative sign of moments has to be agreed upon. In all the following problems, **forces which produce or tend to produce**

rotation of the body acted upon about the point in question in the sense of the motion of the hands of a clock have positive moments.

It may be said here that the other sense of rotation is very often used as the positive. It really does not matter which convention is used **as long as one convention is adhered to throughout the same problem.**

As an illustration, let P and Q (Fig. 15) represent two forces acting on a body, and let A be any point in the body, a and b being the perpendicular distances of P and Q, respectively, from the point A.

Imagining the body to be pinned at A, the force P would cause or tend to cause a rotation of the body in the counter clockwise sense of motion. The moment of P about A is, therefore, negative, and may be concisely written

$$M_P = -P.a.$$

Under the same conditions, Q would cause or tend to cause a rotation of the body in the same sense as the motion of the hands of a clock. Q, in consequence, is said to have a positive moment, so that

$$M_Q = Q.b.$$

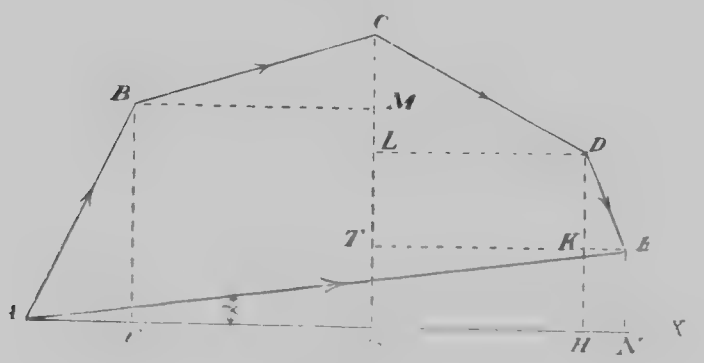


Fig. 16.

Analytical Determination of the Resultant.

To show that $\Sigma X = X_R$ and $\Sigma Y = Y_R$.

Let ABCDE (Fig. 16) be the Vector Polygon of a set of forces, the resultant of which it is required to find [illegible]

Through A draw AX to represent the horizontal direction, and from B, C, D, and E drop perpendiculars to AX, intersecting AX at F, G, H, and N, respectively. Through B, D, and E draw BM, DL, and KE parallel to AX, intersecting CG and DH at M, L, and K, respectively.

The horizontal and vertical resolved parts of the forces represented by AB, BC, CD, and DE are represented by the following lines:

$$X_{AB} = AF \qquad Y_{AB} = FB = GM.$$
$$X_{BC} = BM = FG. \qquad Y_{BC} = MC$$
$$X_{CD} = LD = GH. \qquad Y_{CD} = LC.$$
$$X_{DE} = KE = HN. \qquad Y_{DE} = KD = -TL.$$

Denoting the algebraic sum of the horizontal resolved parts by ΣX and the algebraic sum of the vertical resolved parts by ΣY

$$\Sigma X = X_{AB} + X_{BC} + X_{CD} + X_{DE}.$$
$$= AF + FG + GH + HN.$$
$$= AN.$$

But $AN = X_{AE}$ = horizontal resolved part of the resultant; i.e., $\Sigma X = X_R$.

$$\Sigma Y = Y_{AB} + Y_{BC} + Y_{CD} + Y_{DE}.$$
$$= GM + MC + LC - TL.$$
$$= GT = NE.$$

But $NE = Y_{AE}$ = vertical resolved part of the resultant; i.e., $\Sigma Y = Y_R$.

If the inclination of the line of action of the resultant AE to the horizontal be denoted by α, then—

$$\tan \alpha = \frac{NE}{AN} = \frac{\Sigma Y}{\Sigma X}$$

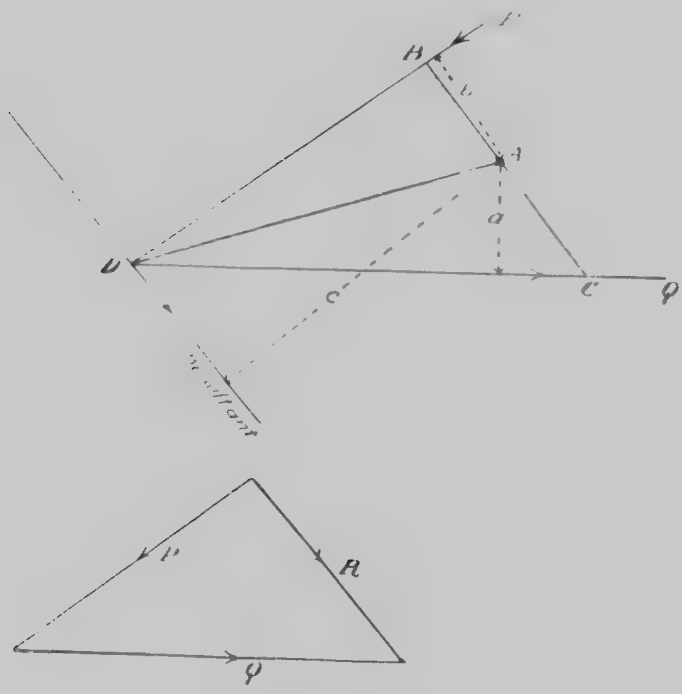
and $R = AE = \sqrt{AN^2 + NE^2} = \sqrt{\Sigma X^2 + \Sigma Y^2}$

To show that $\Sigma M = M_R$.

Consider any two forces, P, Q, as in Fig. 17.

Determine, by means of the vector polygon, the resultant R of P and Q. This resultant will act at D, the intersection of the lines of action of P and Q.

Choose any point A about which to take moments, join A to D and through A draw a line parallel to the line of action of the resultant R, intersecting P and Q at B and C, respectively. The perpendicular distances of A from the lines of action of P, Q, and R are b, a, and c, respectively.



Vector Polygon

Fig. 17

Because the lines BD, DC, and BC are situated closely to one another, as are the directions of P, Q, and R, the triangle BDC may be taken as a vector

polygon of the forces P, Q, and their resultant R, so that

$$P = BD;\ Q = DC;\ R = BC.$$

Taking moments about A

$$\begin{aligned} M_P &= -P.b = -BD.b = -2\ \Delta\ ABD. \\ M_Q &= -Q.a = -DC.a = -2\ \Delta\ ADC. \\ M_R &= R.c = BC.c = 2\ \Delta\ BDC. \end{aligned}$$

Therefore, the algebraic sum of the moments of the forces P and Q is ΣM

$$\begin{aligned} \Sigma M &= M_P + M_Q, \\ &= 2\ \Delta\ ABD - 2\ \Delta\ ADC, \\ &= 2\ \Delta\ BDC. \end{aligned}$$

But $2\ \Delta\ BDC = M_R$.

Therefore, $\Sigma M = M_R$.

That is, the algebraic sum of the moments of two forces about any point is equal to the moment of their resultant about the same point.

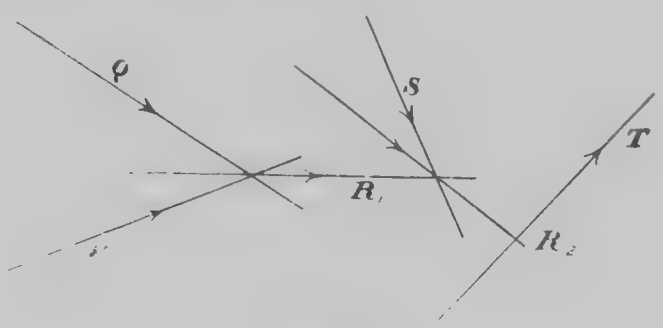


Fig. 18.

That the proof of the last proposition will apply to a case where the set of forces is more than two in number may be seen by considering a set of forces such as P, Q, S, and T represented in Fig. 18. P and Q may be replaced by their resultant R_1, so that R_1, S, and T are equivalent to the original set of forces. R_1 and S may be replaced by their resultant R_2, leaving R_2 and T equivalent to the original four forces; that is, any number of forces comprising a set may be finally reduced to an equivalent pair of forces by such a system of combining and substituting resultants, and to the final pair

so found may be applied the preceding proof. **Therefore, $\Sigma M = M_R$ for any set of forces.**

In Fig. 17, the point A might have been taken in several other positions relative to P, Q, and R. If the point be taken in the line of action of either P or Q, the moment of either P or Q, respectively, would be zero. If the point be taken in the line of action of R, it is evident that a new construction must be devised. In this case, draw through the point a line parallel to either the force P or Q, producing it to intersect the line of action of the remaining force. The triangle so formed may be taken as a vector polygon as before and the same proof applied, the moment of R being zero.

Tabulating these analytical results:

$$\Sigma X = X_R. \qquad \Sigma Y = Y_R. \qquad \Sigma M = M_R. \qquad \tan \alpha = \frac{\Sigma Y}{\Sigma X} \qquad R = \sqrt{\Sigma X^2 + \Sigma Y^2}.$$

From these, the magnitude, direction, sense, and location of the resultant of any set of forces may be fully determined.

Analytical Conditions of Equilibrium.

It has previously been shown that a set of forces in equilibrium can have no resultant. From this it follows that since the value of the resultant is zero, the X, Y, and M of R must be equal to zero, and consequently:—

$$\Sigma X = 0.$$
$$\Sigma Y = 0.$$
$$\Sigma M = 0.$$

are the analytical conditions of equilibrium.

Couples.

Definition: A couple is a pair of equal forces which act in parallel directions with opposite senses.

Fig. 19 represents a couple. The two forces are of magnitude P, and are a distance *a* apart.

If the vector polygon be drawn for these two forces it must necessarily close.

Let α be the angle that the directions of the forces make with the horizontal.

Applying the analytical conditions ΣX, ΣY, and ΣM,

$$\Sigma X = P \cos \alpha - P \cos \alpha = 0.$$
$$\Sigma Y = P \sin \alpha - P \sin \alpha = 0.$$

Taking moments about any point A distant x from one of the forces,

$$\Sigma M = P \quad x \quad P x = P.a.$$

If A be taken in any other position, it is found that the result is always the same, and since P and a are both constants,

$$\Sigma M = P.a = \text{Constant}.$$

Summary:

For a couple, then, the vector polygon must close.

$$\Sigma X = 0.$$
$$\Sigma Y = 0.$$
$$\Sigma M = P.a = \text{Constant}.$$

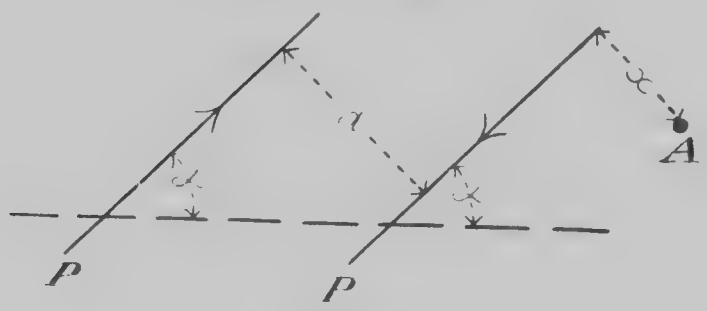


Fig. 19.

Three Forces in Equilibrium.

Three forces in equilibrium must have lines of action which intersect at a common point.

If it were possible, let P, Q, and S (Fig. 132) be three forces in equilibrium. Take moments about the point O, the intersection of the lines of action of P and Q

$$\Sigma M = M_P + M_Q + M_S.$$
$$P.o + Q.o + S.a.$$
$$S.a;$$

i.e., these forces cannot be in equilibrium, for ΣM must equal zero for a set of forces in equilibrium.

It is evident, however, that if the distance a becomes zero, that is, the line of action of S passes through O, that ΣM would also become equal to zero and the three forces would be in equilibrium, provided, of course, that the other conditions, $\Sigma X = o$ and $\Sigma Y = o$, were also fulfilled.

It may be mentioned here that three forces do not have to act at a point in order that ΣX and ΣY be equal

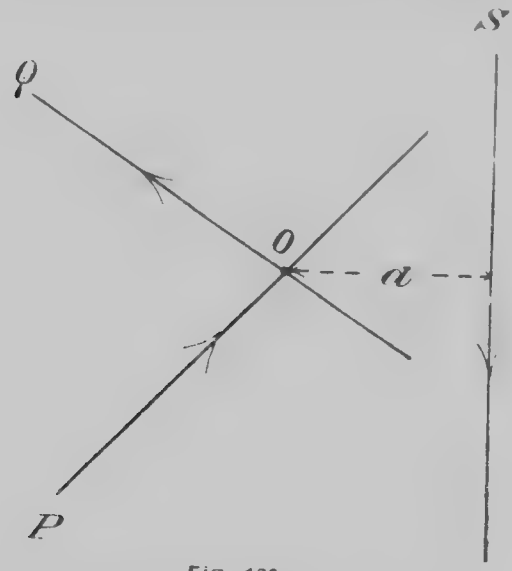


Fig. 132.

to zero. It is merely that ΣM may equal zero for three forces that the above condition must be fulfilled.)

The reader must clearly understand that every set of three forces which acts at a point is not in equilibrium: but if three forces are in equilibrium they must fulfil the condition enunciated at the beginning of this proposition.

CHAPTER V

DETERMINATION OF UNKNOWN FORCES TENSION AND COMPRESSION STRESS IN TRUSS MEMBERS.

Let Fig. 20 represent a plank resting on an abutment A such that 10 feet and 30 feet of the total length overhang to the left and right, respectively. If 100 pounds be placed on the left hand end of the beam, what must be the magnitude of the force P exerted at the other end in order to preserve equilibrium, and what is the magnitude of the abutment reaction A?

Consider the beam as a rigid body. The forces acting on the body are: The abutment reaction, acting vertically upward with a magnitude of A pounds, and the two forces of magnitude, 100 and P pounds, acting

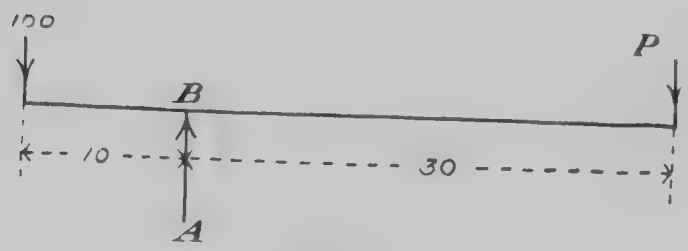


Fig. 20.

vertically downward, and these three forces are in equilibrium; therefore, $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$.

Apply these analytical conditions of equilibrium:

$\Sigma X = X_{100} + X_A + X_P = 0.$

$0 + 0 + 0 = 0$ since all the forces act vertically.

$\Sigma Y = Y_{100} + Y_A + Y_P = 0.$

$-100 + A - P = 0.$

$A - P = 100 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

Take moments about any point in the abutment, e.g., B. The convention as to positive and negative moments must be kept in mind.

[illegible]

Abutment Reactions.

Fig. [illegible] represents a truss supported on two abutments, [illegible] to find the value of the abutment reactions AE and DC.

Let the unknown values of AE and DC be P and Q, respectively.

The truss is acted upon by a set of outside forces which are in equilibrium; therefore, for this set of forces, $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$.

Take moments about any point in the line of action of the other abutment reaction, say, DC.

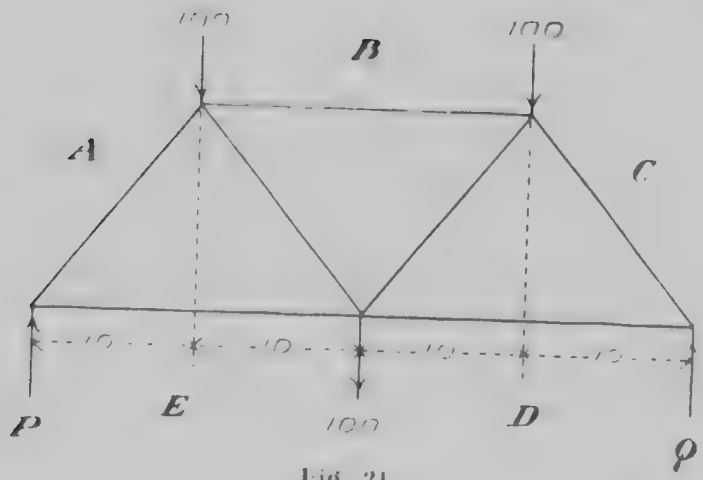


Fig. 21.

[illegible] the abutments support all the load, and [illegible] the foundation support [illegible] pounds, then [illegible] It is [illegible] the student, however, to see that this reasoning [illegible] really based upon the condition $\Sigma Y = 0$.

Tension and Compression.

If a steel bar AB (Fig. 22) rest against a wall and a resultant force P be exerted at B against the bar, the bar is said to be **in compression.**

Keeping in mind Newton's third law of motion, that action and reaction are equal and opposite, it is evident that at B there must be a force exerted contrary and equal to P. This is indicated by D. If any section such as C be examined, it also follows that the right hand particles must be exerting a force equal to P, which is resisted by the left hand particles, as indicated by E and F. Arriving at the end A, there is exerted a push against the wall of magnitude P as represented by G.

in tension

If at any section of a body it is found that the resultant of the forces acting at the section acts against the section, the body is said to be in compression; or, if this resultant is found to be acting away from the section, the body is in tension.

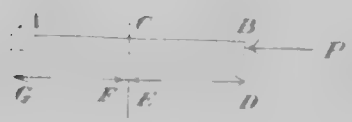


Fig. 22.

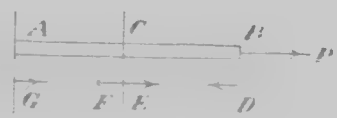


Fig. 23.

The Stress in Truss Members.

[illegible] The truss chosen to illustrate this is known as [illegible] bridge building the world over. The example given is [illegible] thy a very simple case, but, while difficulties may [illegible] in other problems relating to stress in truss members, they are found to be due to lack of knowledge as to where to begin the problem rather than to any lacuna in the principle involved. Experience in working problems is the only remedy for this last-named obstacle.

Graphical Solution.

[illegible]

[illegible] AB

[illegible] AB and CA,

[illegible]

Such forces as AB and CA will always be referred to as unknown forces.

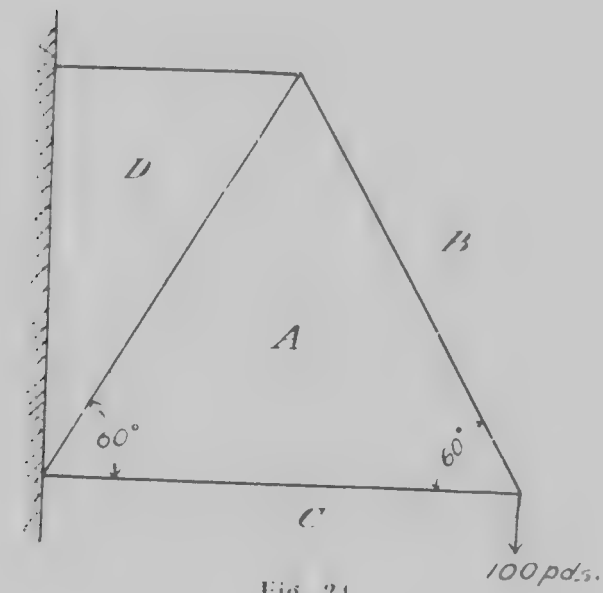


Fig. 24.

From any initial point B (Fig. 26) draw a line parallel to BC in Fig. 25 to represent the direction of the force BC. Cut off BC, in Fig. 26, to represent a magnitude of 100 pounds, placing the sense mark from B to C, thus totally representing the force BC.

From C (Fig. 26) draw a line parallel to CA in Fig. 25. Now, since the magnitude of the force CA is

[illegible]

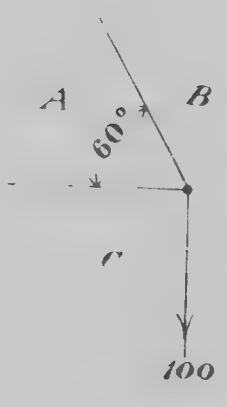


Fig. 25.

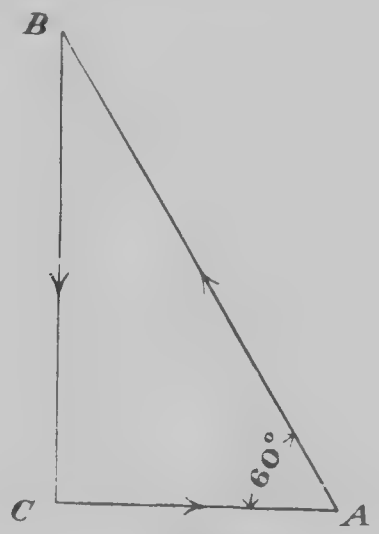


Fig. 26.

that CA and AB Fig. [illegible] represent completely the hitherto unknown forces CA and AB, their senses being such as to give continuous sense from the initial point to the final point of the polygon as indicated.

From the data on the statical diagram the triangle BCA is a 60° right-angled triangle. It follows that since BC represents 100 pounds, CA and AB represent $\frac{100}{\sqrt{3}}$ and $\frac{200}{\sqrt{3}}$ pounds, respectively.

Placing the values of CA and AB as found in the vector polygon, on the statical diagram, it is seen that

[illegible] the point [illegible] AB [illegible] the point [illegible] CA [illegible] and AB [illegible]

[illegible] pounds and [illegible] pounds [illegible] For the reasoning on tension and compression see page 34.)

[illegible] diagram for the point ABD. Since the member AB is in tension, there is a known [illegible] AB acting away from the point ABD as indicated. The two forces BD and DA are unknown.

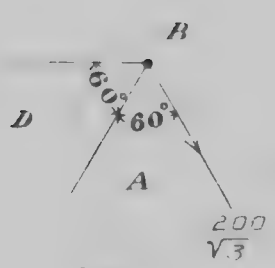


Fig. 27.

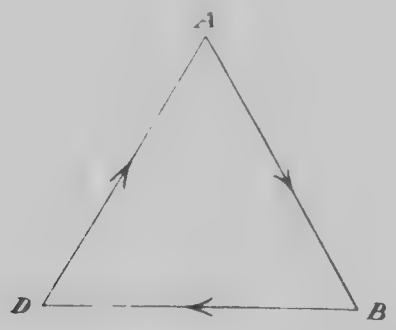


Fig. 28.

Referring to Fig. 28, AB represents the known force AB. From B is drawn a line to represent the direction of the unknown force BD, and from A a line [illegible] drawn to represent the direction of the second unknown force DA. These two lines intersect at D. BD and DA represent the unknown forces BD and DA, and because the triangle ABD is equilateral (from data), BD and DA must each be equal to AB; i.e., of $\frac{200}{\sqrt{3}}$ pounds magnitude. BD acts away from the point ABD and DA against the point. The members BD and DA are, therefore, in tension and compression, respectively.

The results of the preceding solution are given in Fig. 29, compression and tension members being shown by thickened and light lines, respectively.

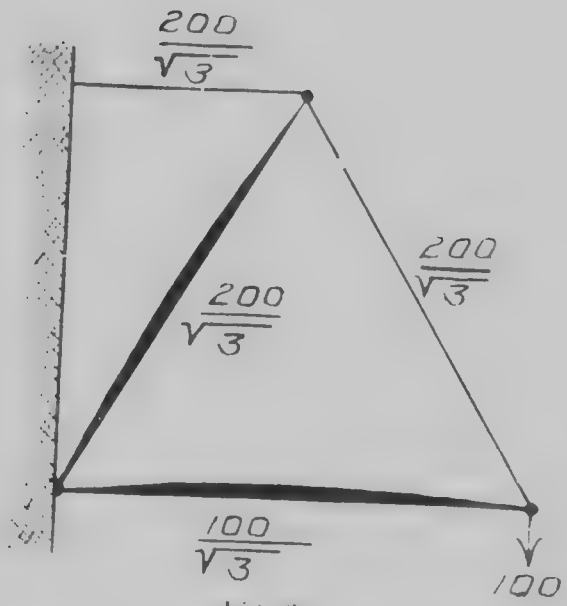


Fig. 29

Analytical Solution.

[illegible]

[illegible]

$DA = AB$ [illegible]

$BD = AB + DA \cos$ [illegible]

$BD =$ [illegible] pounds.

[illegible] compression and [illegible] The student is advised to work [illegible] ABC.

[illegible] check [illegible] noticed that [illegible] ΣM was [illegible] here all three [illegible] experience [illegible] greater [illegible] Nor is [illegible] can be [illegible] equations [illegible] but it will soon be [illegible] equations [illegible] often be avoided.

CHAPTER VI

THE EQUILIBRIUM OR FUNICULAR POLYGON.

The Vector Polygon gives the magnitude, direction, and sense of the Resultant of a set of forces, but does not determine the true position of the line of action of this Resultant. The problem of locating the Resultant may be accomplished by means of what is termed the Equilibrium Polygon.

Let P, Q, and S (Fig. 149) be any set of co-planar forces, the Resultant of which it is required to locate.

In a previous discussion, it was shown that the size and shape of the body acted upon can in no way alter the position of the line of action of the Resultant and Equilibriant of a set of forces; in other words, **the Resultant and Equilibriant are independent of the size and shape of the body acted upon.** This being the case, it will be shown that the position of the Equilibriant and Resultant may be determined by replacing the original body by an attached, free-jointed frame, the shape of which will give the desired information.

ABCD (Fig. 150) is a Vector Polygon for the forces P, Q and S. DA evidently represents the Balancing Force or Equilibriant of this set of forces.

[illegible] if the Equilibriant can be located, the position of the Resultant will also be known.

[illegible]

Choose any point O [illegible] B, C, and D [illegible] P.

[illegible] BO, and OA form a closed [illegible]

[illegible] For instance, the forces BO and OA [illegible] P.

[illegible] the line of action of P (Fig. [illegible]) [illegible] forces OA and BO. These three forces [illegible]

[illegible] time being, any discussion as to how these [illegible] OA and BO are exerted and what constitutes the [illegible] by the three forces, P, BO, and OA, [illegible]

Produce the line of action of BO to intersect the line [illegible] Q at X, and at this point introduce the forces represented by CO and OB (Fig. 150). The line of action [illegible] OB evidently coincides with BO (Fig. [illegible]). It is seen

[illegible] e forces OB and CO acting at X [illegible] force Q.

Produce the line of action of CO [illegible] S at Y. At Y introduce the forces DO [illegible] represented by the lines DO and OC [illegible]

Produce the line of action of DO [illegible] of action of OA [illegible] es, AO and OD, represented by AO and OD [illegible]

Let the polygon WXYZ [illegible] W, X, Y, [illegible] will be in equilibrium if some force be intro- [illegible] AO and OD [illegible]

[illegible] P, Q, S, and T [illegible] X, Y, [illegible] it is seen that the force T [illegible] P, Q, and S; i.e., the force T [illegible] P, Q [illegible] S, [illegible] which [illegible] ired Resultant.

The polygon WXYZ (Fig. 149) is known as an Equilibrium or Funicular Polygon.

The Balancing Couple.

[illegible] P, Q, [illegible] S [illegible] CO [illegible] OA [illegible] OA [illegible] AO [illegible]

[illegible] S, Q, [illegible] CO, OA, [illegible]

FIG. 32.

[illegible]

Balancing Couple

Resultant Couple.

The Resultant [illegible] Balancing Couple.

Second Graphical Condition of Equilibrium.

[illegible] HE were coincident that the frame, which would [illegible] form DEF, would hold the forces S, Q, and P in equilibrium. The forces S, Q, and P [illegible]

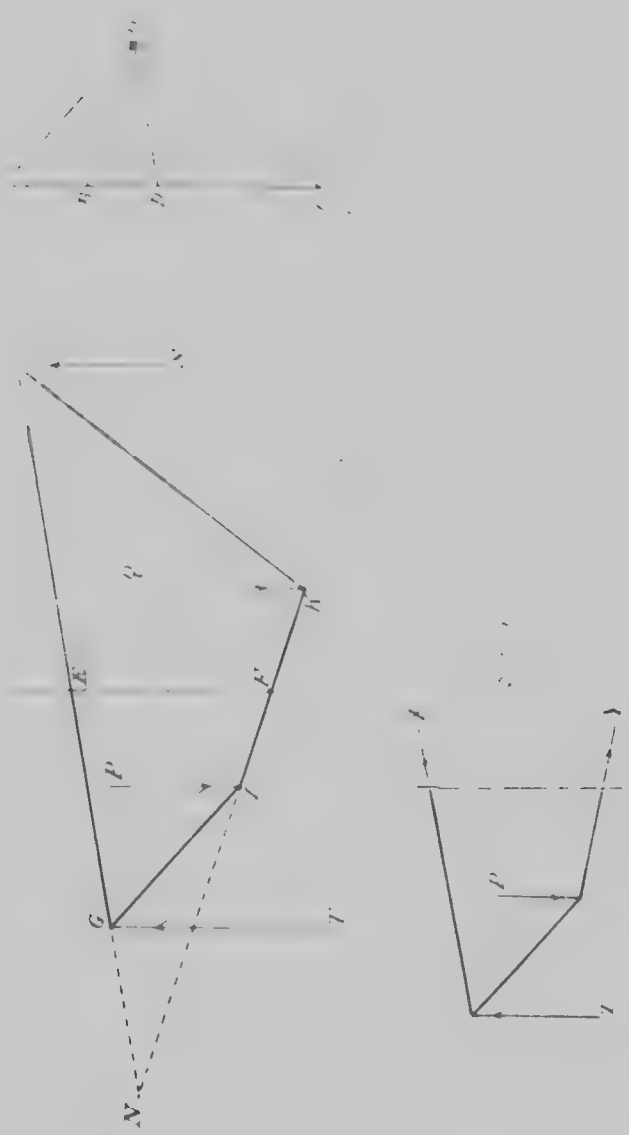
If, then, the [illegible] of forces must be in [illegible] concisely stated by saying that **the Equilibrium or Funicular Polygon must close**, bearing in mind the [illegible]

Graphical and Analytical Determination of the Resultant.

Graphical.	Analytical
Vector Polygon gives the magnitude, direction, and sense of the Resultant.	$\Sigma X = X_R$ $\Sigma Y = Y_R$ $R = \sqrt{\Sigma X^2 + \Sigma Y^2}$. $\tan \theta = \frac{\Sigma Y}{\Sigma X}$
Equilibrium Polygon locates the line of action of the Resultant	$\Sigma M = M_R$

Conditions of Equilibrium.

Graphical.	Analytical
Vector Polygon must close.	$\Sigma X = 0$.
Equilibrium Polygon must close.	$\Sigma Y = 0$.
	$\Sigma M = 0$.



P, Q, S, and T are held in equilibrium by the jointed forces of Equilibrium Polygon GHKI

. . the forces exerted on one side of the section by the cut members hold in equilibrium the external forces acting on the portion of the frame lying to the other side of the section.

Also that, the resultant of the external forces to one side of the section acts through the intersection of the cut members produced.

CHAPTER VII

THE METHOD OF SECTIONS.

It is often necessary to find the stress in one or more members of a truss, irrespective of the stress in other members. To obviate the tedious operation of working from point to point till the stress in the required member is finally reached, what is known as the **Method of Sections** is used, whereby the stress, say, in EF Fig. 33, may be arrived at immediately **from a consideration of the forces acting on the truss.**

Imagine the truss represented in Fig. 33 as cut into

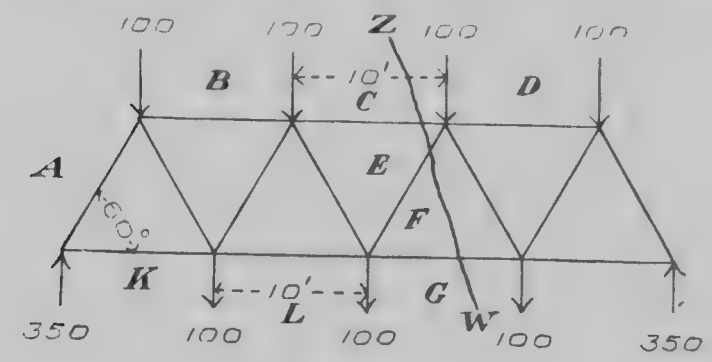


Fig. 33.

two sections at ZW. Consider the portion of the truss to the left of ZW. This considering the portion to the left is purely arbitrary. The same results would eventually be reached by considering the right-hand section, but in order to avoid confusion, the convention of considering the left-hand portion is agreed upon.

This portion of the truss to the left of ZW is held in equilibrium by certain forces **acting on it.** There is no difficulty in seeing that the left-hand abutment reaction AK, and the four loads AB, BC, KL, and LG, act on the body to left of ZW. **But that these forces alone could not preserve equilibrium** is easily seen. There must, then, be other forces acting on the body, and the only possible places where forces could act is where the portions of the members CE, EF, and FG, to the right of ZW, touch the portions to the left.

As far as the forces acting on the body are concerned, it does not matter whether the body be con-

sidered as a truss form or as a solid body having the boundary outline of the truss. It is generally less confusing, however, to choose the latter method, because it seems to concentrate attention on the extraneous forces.

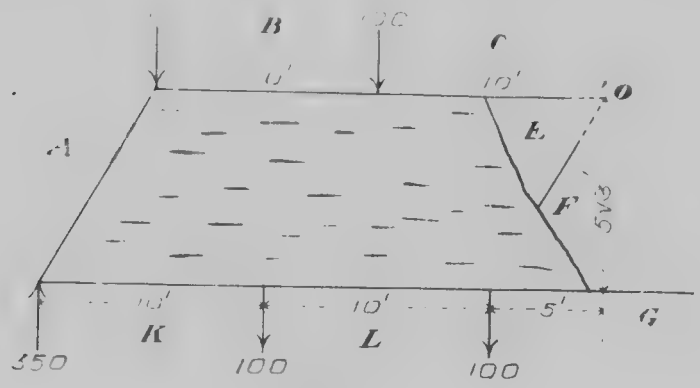


Fig. 34.

Fig. 34 represents the forces acting upon the portion of the truss to the left of ZW, considered as a solid rigid body. These forces are in equilibrium, and, since there are only three unknown forces, CE, EF, and FG, it is possible to solve for them by means of the three analytical equations of equilibrium, $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$.

$$\Sigma Y = Y_{AK} + Y_{KL} + Y_{LG} + Y_{GI}$$
$$Y_{EF} + Y_{LC} + Y_{CB} + Y_{BA} = 0.$$
$$350 - 100 - 100 = 0.$$
$$FE \sin 60^\circ + 0 - 100 - 100 = 0.$$
$$350 - 400 + FE \frac{\sqrt{3}}{2} = 0.$$
$$FE = \frac{100}{\sqrt{3}}$$

From the positive sign of the result, the Y_{EF} must be positive.

The member FE is, therefore, in tension $\frac{100}{\sqrt{3}}$ pounds.

Take moments about the intersection of CE and EF at the point O.

$$\Sigma M \quad M_{AK} + M_{KL} + M_{LG} + M_{GI}$$
$$M_{FE} + M_{EC} + M_{CB} + M_{BA} = 0.$$
$$350.25 - 100.15 - 100.5 + GF.5\sqrt{3}$$
$$0 + 0 - 100.10 - 100.20 = 0.$$
$$350.25 - 100(15 + 5 + 10 + 20)$$
$$+ GF.5\sqrt{3} = 0.$$
$$GF = \frac{750}{\sqrt{3}}$$

From the negative sign of the result the moment of GF about O must be negative. GF, therefore, acts away from the section;

i.e., the member GF is in tension $\frac{750}{\sqrt{3}}$ pounds.

$$\Sigma X \quad X_{AK} + X_{KL} + X_{LG} + X_{GI}$$
$$X_{FE} + X_{EC} + X_{CB} + X_{BA} = 0.$$
$$0 + 0 + 0 + GF + FE.\cos 60^\circ$$
$$CE + 0 + 0 = 0.$$
$$\frac{750}{\sqrt{3}} + \frac{100}{\sqrt{3}}\cdot\frac{1}{2} + CE = 0.$$
$$CE = -\frac{800}{\sqrt{3}}.$$

From the negative sign of this result, the X_{CE} must be negative. CE, therefore, acts against the section;

i.e., the member CE is in compression $\frac{800}{\sqrt{3}}$ pounds.

The student is advised to choose other sections in the same truss and work out the stresses in the various members by this means.

It must be recognized, however, that the Method of Sections is not a method particularly to be applied to certain problems, but is, in reality, used in all statical problems. Whenever one considers a set of forces acting on a body, a section has been taken around some portion of material which may even be part of a larger body, and this isolation of a body acted on constitutes the method of sections.

CHAPTER VIII

THE BEAM.

In the following discussion, the [illegible] supported [illegible] on two abutments, [illegible] each extremity [illegible] will be considered. This type of beam is [illegible] commonly met with in engineering [illegible] cases do arise in which a beam [illegible] upon more than one abutment, or in [illegible] supports are not free, as in the case [illegible] extremities built into the supporting [illegible]. It [illegible] to point out that these last cases [illegible] be investigated by the theory of these pages [illegible] a knowledge of the laws of the [illegible] material and of the calculus.

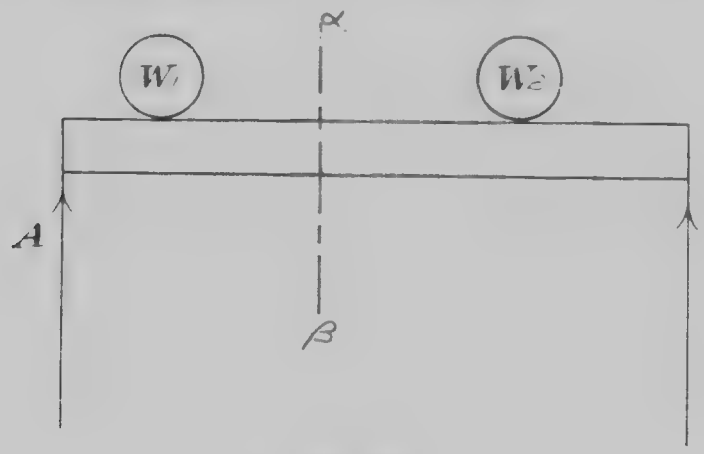


Fig. 35.

Let [illegible] represent a beam supported [illegible] carrying [illegible] loads, W_1 and W_2. [illegible] reaction may be [illegible] A, [illegible] value will depend [illegible] distances [illegible] W_1 and W_2 [illegible] the portion of the beam [illegible] the Method of Sections. [illegible] known [illegible] portion to the [illegible] These forces [illegible] It is evident, [illegible]

upon the portion of the beam being considered. The only possible place where such a force or forces could act is at the section $\alpha\beta$ in the beam. Let Fig. 36 represent the condition of affairs as far as the forces acting upon the body to the left of $\alpha\beta$ are concerned. These forces are in equilibrium; therefore, ΣX, ΣY, and ΣM must each equal zero.

$$\Sigma Y = Y_A - Y_{W_1} + (Y\text{'s of unknown forces}) = 0.$$
$$A - W_1 + (Y\text{'s of unknown forces}) = 0.$$

Replacing the Y's of the unknown forces by their resultant, which may be denoted by V

$$A - W_1 + V = 0.$$
$$V = (A - W_1).$$

From which it is seen that V is equal in magnitude but acts in the opposite sense to the algebraic sum of

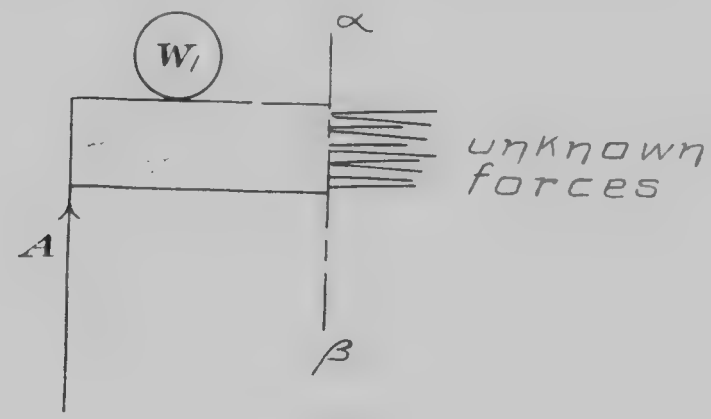


Fig. 36.

the vertical resolved parts of the exterior forces to the left of $\alpha\beta$; i.e., V resists $A - W_1$. This quantity $(A - W_1)$ is called the **Vertical Shearing Force** at the section under consideration.

Definition:

The Vertical Shearing Force at any section is equal to the algebraic sum of the vertical resolved parts of the exterior forces acting on the body to the left of the section being considered.

This is a general definition, but since the loads and abutment reactions in most cases, and certainly in all

cases that will be taken up here, act in the vertical direction, a more concise definition may be employed.

The Vertical Shearing Force for Vertical Loading and Abutment Reactions is equal to the algebraic sum of all exterior forces to the left of the section being considered.

Applying ΣX to the forces acting as indicated in Fig. 36

$$\Sigma X = X_A + X_{W_1} + (X\text{'s of unknown forces}) = 0.$$
$$0 + 0 + (X\text{'s of unknown forces}) = 0.$$
$$(X\text{'s of unknown forces}) = 0.$$

Now, there are two ways in which the algebraic sum of the X's of the unknown forces may be equal to zero. Either all the X's are equal to zero, or the sum of the positive X's is equal in magnitude to the sum of the negative X's.

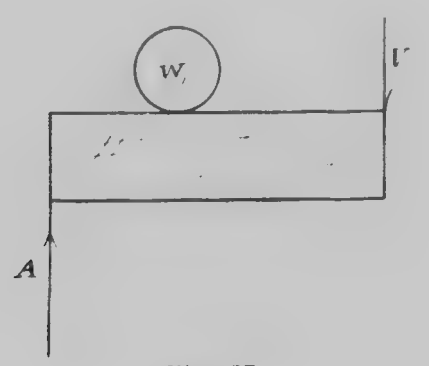


Fig. 37.

In order to determine which of these two cases is the true one, consider the forces acting on the portion of the body to the left of $\alpha\beta$. Replace the unknown forces shown in Fig. 36 by their X's and Y's. The Y's of these unknown forces have been shown to be equivalent to a force V, which will act downward if W be taken as less than A. If now we consider the first case that all the X's are equal to zero, the state of affairs would be as represented in Fig. 37. It is seen at once that such a condition is impossible, since the forces

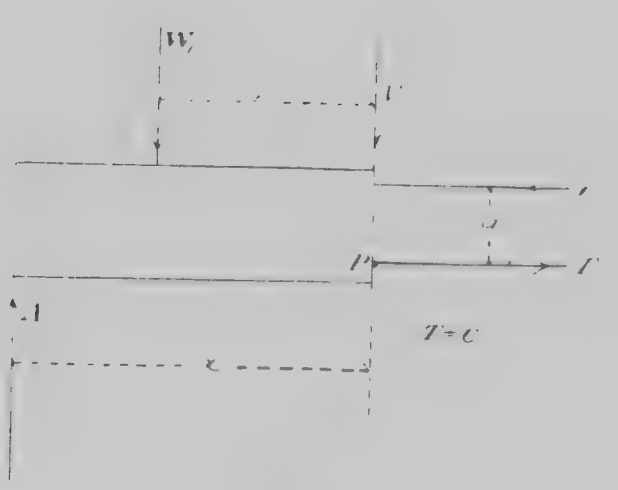


Fig. 38.

Bending Moment

[illegible]:

The Bending Moment at any section for vertical loading and abutment reactions is equal in magnitude to the algebraic sum of the moments of the exterior forces to the left of the section, about a point in the section.

CHAPTER IX

VERTICAL SHEARING FORCE AND BENDING MOMENT FOR STATIONARY LOADS ON A BEAM.

The student is particularly advised to memorize the definitions of the Vertical Shearing Force (V.S.F.) and the Bending Moment (B.M.) at any section in a beam as given in the last chapter.

In explanation of the meaning of these terms, V.S.F. and B.M., it may be pointed out that in a beam under a load there is a tendency to bend; and because the deflection due to bending is different at various sections, it is reasonable to suppose that the bending influence is variable for different sections of the beam, which reasoning is borne out by the theory. The B.M.

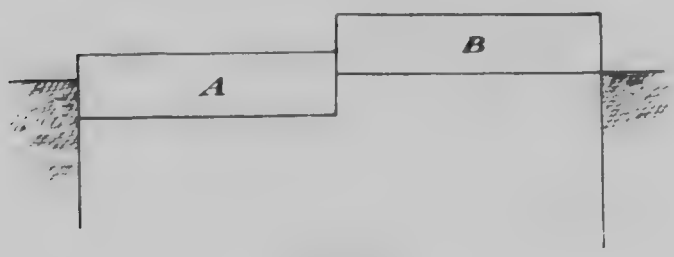


Fig. 39.

is related to this bending influence. Secondly, any portion of a beam has a tendency to slip past, or **shear** past, the other portion of the beam. For instance, in Fig. 39 the portion A has sheared past B. Whether any given portion of a beam actually does shear or not is aside from the question that there exists such a tendency. The V.S.F. is related to this tendency, which is variable for different sections.

To Trace the Variation in V.S.F. Due to a Single Concentrated Load.

In Fig. 40, consider **any** section between the load W and the left-hand abutment. The V.S.F. at any such section algebraic sum of exterior forces to left of section.

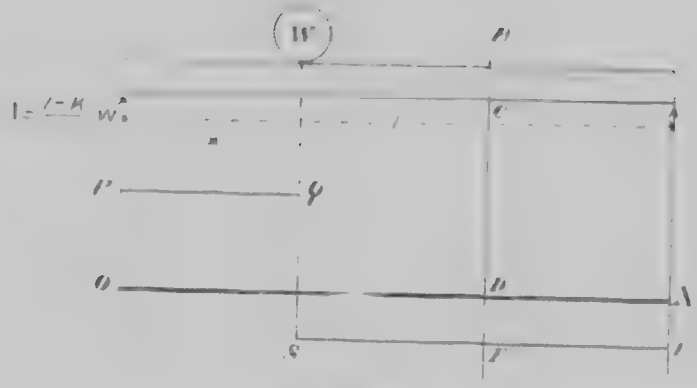


Fig. 40.

[illegible]

Consider **any** section to the right of W [illegible]

Therefore V.S.F. for all sections to right of W [illegible] W > A [illegible]

Let OX [illegible]

Plotting these ordinates with OX [illegible] PQ and ST [illegible] OX [illegible] The diagram PQST is known as the **V.S.F. Diagram.** If it is desired to find the V.S.F. at [illegible]

To Trace the Variation in V.S.F. Due to any Number of Concentrated Loads

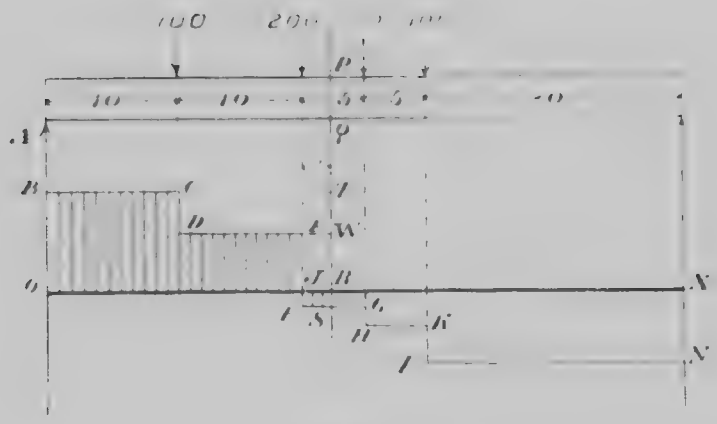


Fig. 41.

In Fig. 41 the left-hand abutment [illegible]

$V = 4.5.100 + 3.5.200 + \frac{1}{2}.50 \ldots$

$= 265$ pounds.

Let y = V.S.F. at any section.

Between A and first load, $y = A = 265$

Between first and second load, $y = 265 - 100 = 165$

Between second and third load,

$y = 265 - 100 - 200 = 35.$

Between third and fourth load,

$y = 265 - 100 - 200 - 50 = 85$

Between fourth load and right-hand abutment,

$y = 265 - 100 - 200 - 50 - 100 = -185$

Plotting these ordinates with OX as base line a V.S.F. diagram of the form BC, DE, FG, HK, is obtained, such that the ordinate to the diagram immediately beneath any section of the beam represents the value of the V.S.F. at that section.

It will be noticed that **the change in value of shearing force** from a section immediately to the left of any load to a section just to the right of the load **is equal to the magnitude of the load itself.** For instance, in the last numerical case, the changes represented by CD, EF, GH, and KL are each equal in magnitude to the loads immediately above these lines.

Representation of Bending Moment at any Section of a Beam.

Let PQ (Fig. 41) be the section, the B.M. at which it is required to represent. PQ produced intersects the V.S.F. diagram in RS.

Let PQ be three feet to the right of the 200-pound load.

Produce BC and DE to intersect PQ at T and W.

B.M. at PQ	$A \times 23 - 100 \times 13 - 200 \times 3$.
But $A \times 23$	$OB \times BT$ area OT.
100×13	$CD \times CT$ area DT.
200×3	$EF \times EW$ area FW.
Therefore, B.M.	area OT — area DT — area FW.
	area OBCDEJ — area JFSR.
	area of V.S.F. diagram to left of PQ.

It must, of course, be remembered that areas above OX are considered positive and those below OX negative. Keeping this last statement in mind, it is seen that

The area of the V.S.F. diagram to the left of any section represents the B.M. at that section.

The B.M. at any section is always positive, for the positive area of the V.S.F. diagram to the left of any section is, with one exception, always greater than the negative area to the left of the same section. It may be proven geometrically that the positive and negative areas to the left of the right-hand abutment are equal; hence, the B.M. at this abutment is zero. The B.M. at the left-hand abutment is also zero, since there is no area of the V.S.F. diagram to the left of that section.

Section at which Maximum B.M. Occurs for a Single Load.

ABDF (Fig. 42) is the V.S.F. diagram for the load W. Consider areas above OE positive, below OE negative.

The B.M. at any section NP to the left of the load is represented by the area ANPO.

The B.M. at any section GL, to the right of the load, is represented by the area ABCO — CGLD.

Both of these areas are less than the positive area ABCO. Therefore, the area ABCO must represent the greatest B.M.

But ABCO represents the B.M. at a section infinitely close to BC, but still to the left of the load. Practically, this is the section BC. It follows that:

The Maximum B.M. for a single load in a given position occurs at a section directly under the load.

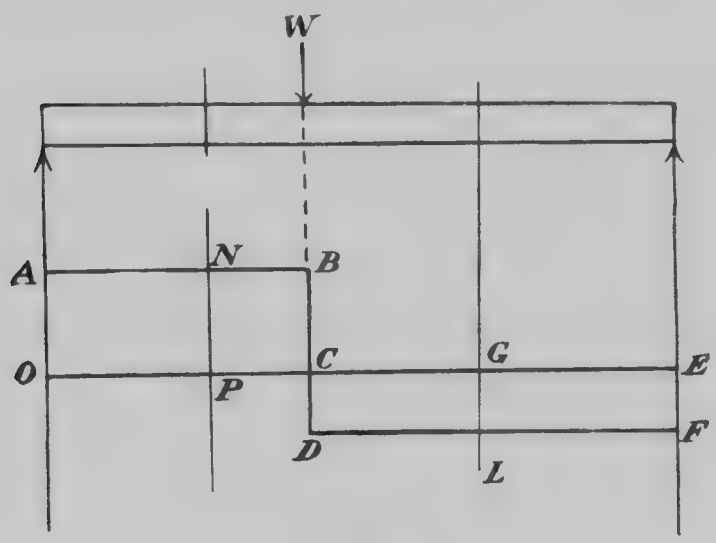


Fig. 42.

Uniformly Distributed Load.

A uniformly distributed load is one such that, no matter how small a portion of the area loaded is considered, the load per **unit area** is always the same.

Since the discussion is confined to co-planar forces, a uniformly distributed load in the following problems will mean a load of so much per **unit length.**

[illegible]

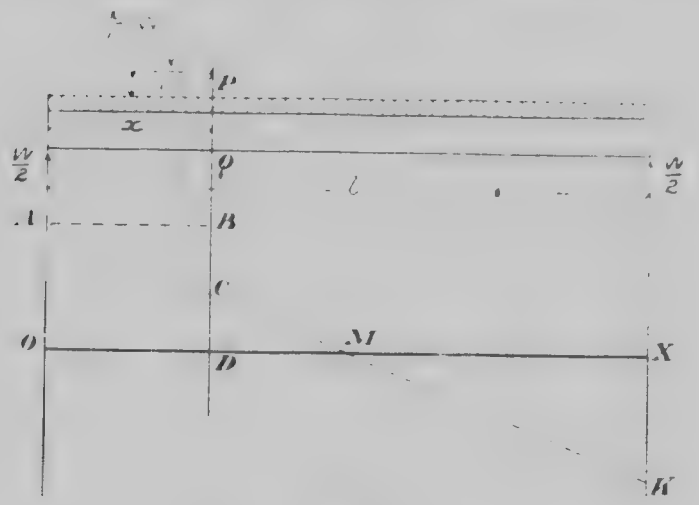


Fig. 43.

To Trace the Variation of V.S.F. and B.M. in a Beam Supporting a Uniformly Distributed Load of W Pounds.

Let the hatched area of Fig. 43 represent the load uniformly distributed over the entire length of the beam.

Total load = W lbs. Abutment reactions = $\frac{W}{2}$.

Load per unit length = $\frac{W}{l}$.

Take **any** section PQ at a distance x from the left-hand abutment.

V.S.F. at PQ = [illegible] left-hand abutment [illegible] load to left of PQ.

Load to left of PQ = $\frac{W}{l}x$.

Therefore, $v = \frac{W}{2} - \frac{W}{l} x$

If the values of v be plotted with OX as axis of x, the curve will be a straight line such that, as in the previous cases, the value of the V.S.F. at any section will be directly under that section. This is shown by the line AK.

To prove the curve, $v = \frac{W}{2} - \frac{W}{l} x$ . . . . . (1)

When $x = 0$, $v = \frac{W}{2} = OA$

When $x = L$, $v = -\frac{W}{2} = AK$.

When $v = 0$, $x = \frac{l}{2} = OM$

From this it is seen that the V.S.F. varies from a positive value of $\frac{W}{2}$ at the left-hand abutment, to a value $\frac{W}{2}$ at the right-hand abutment, being equal to zero at the centre of the beam.

In order to consider the variation in B.M., let the load to the left of **any** section PQ be replaced by an equivalent concentrated load of magnitude $\frac{x}{l}.W$, which will act at a distance $\frac{x}{2}$ from PQ as dotted in Fig. 43.

B.M. at PQ $= \frac{W}{2}.x - \frac{W}{l}.x \cdot \frac{x}{2}$ . . . . . (2.)

Produce PQ to intersect the V.S.F. diagram at C and D. Draw AB perpendicular to PQ.

OD and DC are co-ordinates to a point on AK below PQ

$$OD = x, \quad y = DC$$

Therefore, from (1.), $y = DC = \frac{W}{2} - \frac{W}{1} \cdot x \ldots (3.)$

From the figure it is seen that

$$CB = DB - DC = OA - DC \ldots\ldots\ldots\ldots (4.)$$

But $OA = \frac{W}{2}$

Substitute in (4.) this value of OA, and also the value of DC found in (3.).

Then, $CB = \frac{W}{2} - \frac{W}{2} + \frac{W}{1} \cdot x$.

$$\frac{W}{1} \cdot x.$$

Tabulating these results

$$\frac{W}{2} = OA.$$

$$x = OD = AB$$

$$\frac{W}{1} \cdot x = CB.$$

Substituting these values into (2.)

$$B.M. = OA \times OD = CB \times \frac{AB}{2}$$

Area OB = $\frac{1}{2}$ ABC.

Area OACD

Area of A S T [illegible] section

The B.M. [illegible]

It is seen that **the maximum B.M. for a beam supporting a uniformly distributed load over its entire length is at the centre of the beam,** for the area AMO is the greatest positive area.

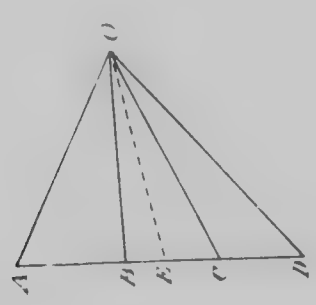


Fig. 44 b.

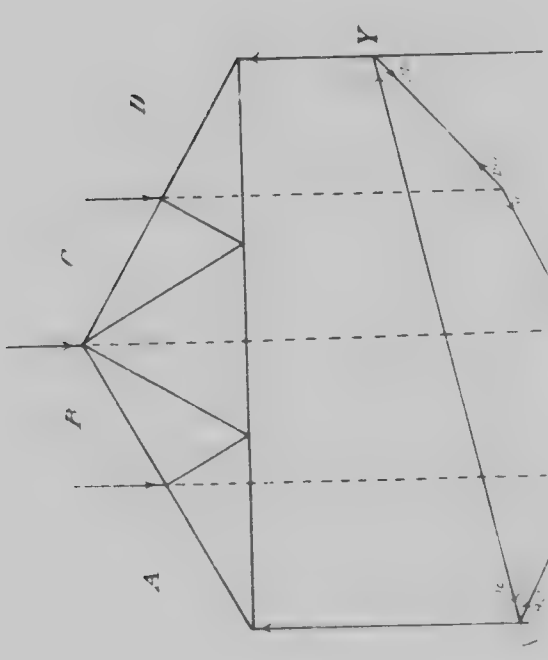


Fig. 44 a.

Graphical Determination of Abutment Reactions.

[illegible] to this, the [illegible] reactions [illegible] calculated analytically. [illegible] method of determining [illegible] follows:

Let [illegible] supporting the loads AB, BC, CD [illegible] These loads, [illegible] DE and EA, [illegible]

The vector polygon, AB, BC, and CD [illegible] since the forces [illegible] the vector polygon [illegible] The only [illegible] reactions. It follows, then, that between D and A [illegible] point E, such that DE and EA [illegible]

[illegible] O to A, B, C, and D [illegible] AB by BO and OA [illegible] balancing BC [illegible] CO and OB, and CD by DO and OC. Produce [illegible] OA and DO [illegible] X and Y [illegible] D and A [illegible] DE and EA [illegible] by EO, OD, and AO, OE [illegible] X and Y [illegible] EO and OE [illegible] XY. This determines the point E, for, in order that the forces EO and OE [illegible] the direction XY, the line joining O to E must be parallel to XY, i.e., **to locate E, through O draw OE parallel to XY, intersecting DA at E.** DE and EA [illegible] reactions.

Relation between Bending Moment and the Funicular Polygon.

Let [illegible] supporting any general [illegible] AB, BC, and CD [illegible] polygon, AB, BC, CD, DE, and EA, the [illegible] DE and EA having been found

graphically by means of the funicular polygon, as explained in the last problem. It is required to find the value of the B.M. at any section (1

Produce (1.) to intersect the funicular polygon at G and K. Let the perpendicular distance of the pole O from AB be Z.

The B.M. at (1.) = Algebraic sum of the moments of EA and AB about any point in (1.).

= Moment of the resultant of EA and AB about any point in (1.).

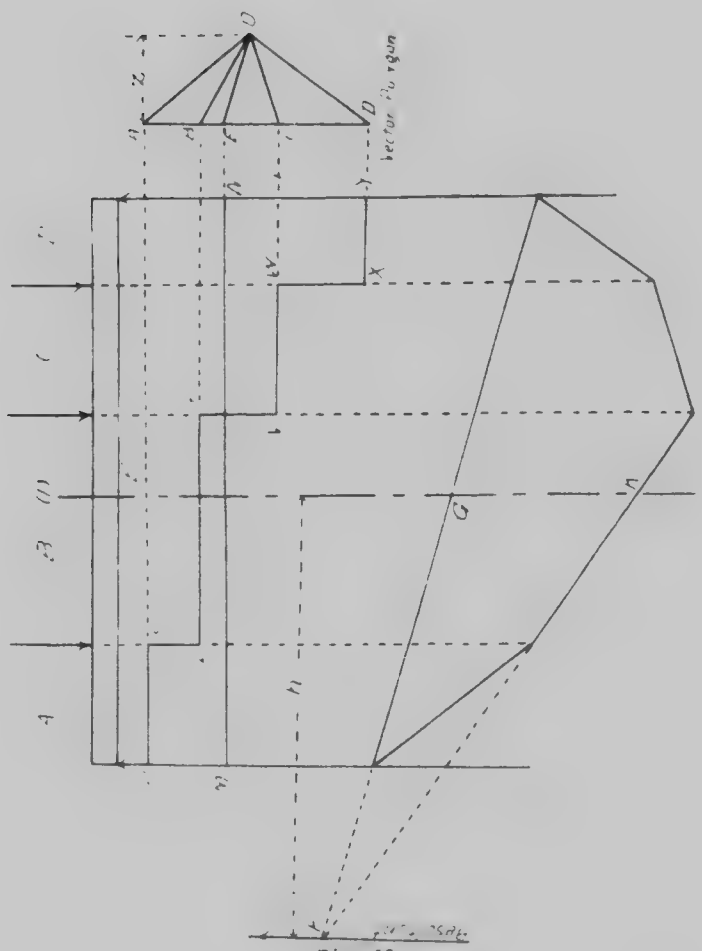


Fig. 45.

To locate the resultant of EA and AB: i.e., the resultant of all forces to the left of plane (1.):

In discussing the properties of the Equilibrium or Funicular Polygon, it was shown that if a section be taken through two members of the frame, the resultant of the external forces to either side of the section acts through the intersection of the cut sides of the polygon. Section (1.) cuts two sides of the polygon at G and K. If, then, the two cut sides be produced to intersect at F, as indicated in Fig. 45, it is evident that the resultant of the forces EA and AB must act through this point. But from the Vector Polygon it is seen that the resultant of EA and AB is represented by EB. Drawing through F a line parallel to EB, the true position of the required resultant is found. The moment of this resultant about a point in section (1.) will give the desired B.M. Let h be the distance of this resultant from section (1.).

Now,

FG = EO.
FK = OB.
GK = BE.

Therefore, Δ's FGK and BEO are similar.

Then,

EB : altitude z.
GK : altitude h.

Or, EB × h = GK × z ..... (3)

But EB represents the resultant of the forces to the left of section (1.).

Therefore, EB × h = B.M. ..... (4)
i.e., from (3.) B.M. = GK × z ..... (5)

Now, GK is called the ordinate of the funicular polygon at section (1.), and it is seen that the B.M. is represented by this ordinate multiplied by a constant quantity z; so that, **provided the proper scale is used, the B.M. at any section is represented by the ordinate of the funicular polygon immediately beneath the section.**

To Find the Scale of B.M.'s to be Used with Ordinates of the Funicular Polygon.

From (4.) in the last discussion

B.M. = EB × h.

(EB in inches × scale of forces) (h in inches × scale of dimensions).

= EB × h (scale of forces × scale of dimensions).

But from (3.) EB × h = GK × z.

Therefore, B.M. = GK (z × scale of forces × scale of dimensions); i.e., the scale to use on GK in order to find the actual value of the B.M. is **(z × scale of forces × scale of dimensions)**, z being taken in inches if the scales are per inch.

Caution:

In order to get a good working scale of B.M.'s, the pole O should be so chosen with reference to AD, that z will be a quantity which will, when multiplied by the scales of forces and dimensions, give an even quantity suitable for use with ordinary scales.

A Graphical Method of Constructing the V.S.F. Diagram for Concentrated Loads.

Let the vector diagram of the forces acting on the beam be drawn to one side and slightly below the representation of the beam as shown in Fig. 45. It will be assumed that the abutment reactions DE and EA have been found. From E draw a line ENM parallel to the longitudinal axis of the beam, and intersecting the abutment reactions at N and M.

The V.S.F. at any section between the left-hand abutment and the first load is equal to the left-hand abutment reaction, the value of which reaction is represented by EA. Therefore, if the scale of shearing forces be chosen the same as the scale of forces used in constructing the vector diagram, the ordinates from MN to the line PS, which is drawn parallel to MN through A, will represent the V.S.F. values at any section between the left-hand abutment and the first load, for these ordinates will each be equal to EA, which is the value of the V.S.F. between the sections indicated.

[illegible] A.S.L. [illegible] AB [illegible]

[illegible] AB [illegible] B

[illegible] to MN [illegible] B, [illegible] from MN to [illegible] A.S.L. [illegible] between the first and second ead. [illegible] of reasoning, [illegible] from MN to AW and XY, which [illegible] to MN [illegible] C and D, respectively, represent the A.S.L. [illegible] the second ead to the right-hand abutment.

CHAPTER X

V.S.F. AND B.M. FOR MOVING LOADS ON A BEAM.

To Trace the Variation in V.S.F. and B.M. for a Single Moving Load.

Let a load of W pounds travel over the beam to a [illegible] the left-hand abutment is represented in Fig. [illegible]. The left-hand abutment reaction A [illegible] W.

RBDH is the V.S.F. diagram for the load in this position.

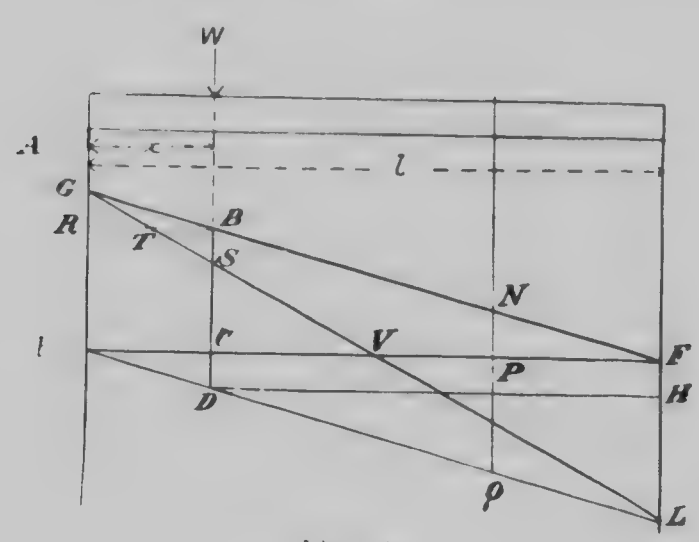


Fig. 46.

Now, since the load is a moving one, it would be extremely inconvenient to draw a series of V.S.F. diagrams for many or all positions of the load.

Looking at the diagram RBDH, which is a perfectly general one, it is seen that because RB and DH are parallel to FL, BC represents the value of the V.S.F. at any plane to the left of the load and CD the V.S.F. at any plane to the right of the load.

But V.S.F. at any plane to left of load $= V$ [illegible] $W = BC$; and V.S.F. at any plane [illegible]

[illegible] $V = W$ [illegible] $W = W = CD$

If, then, ordinates y [illegible] W be plotted [illegible] the load as it moves to different positions [illegible] that y represents the value of the V.S.F. at any plane to the left of the load, and ordinates y' [illegible] $W = W$ be also plotted underneath the load in the same series of positions, so that y' represents the V.S.F. at all planes to the right of the load, the locus of the points so obtained will be the straight lines GF and EL, respectively [illegible] course, be negative for $W >$ [illegible] W.

GFEL is a V.S.F. diagram for the moving load W. Supposing it is desired to find the V.S.F. at a plane NQ: i.e. when the load is over BD, [illegible] when the load is over NQ.

1. The V.S.F. at NQ when the load is over BD is represented by CD, for CD represents the V.S.F. for **all** planes to right of load when load is over BD.

2. When the load is directly over NQ, the V.S.F. at NQ is indeterminate, but NP represents the V.S.F. at all planes to left of NQ, and PQ represents the V.S.F. at all planes to right of NQ. These results may be arrived at analytically by letting x be the distance of the load from the left-hand abutment.

For any plane to the left of the load

$$V.S.F. = x \int_{l}^{l} W$$

For any plane to the right of the load

$$V.S.F. = x \int_{l}^{l} W - W$$

To Find the Maximum Possible B.M. as the Load Moves Over Beam.

[illegible] B.M. [illegible] greatest possible B.M.

Taking the load in a position represented in Fig. [illegible] B.M. for this position is represented by RBCE.

[illegible] GL

The area GSCE is then equal to the area RBCE, [illegible] the area RTSCE is common to both, and the Δ GRT [illegible] Δ TBS.

Therefore, the area GSCE represents the [illegible] B.M. for the load in that position. In the same [illegible] for the load in other positions, the maximum B.M. is represented by the area between GL and EF to the left of the section under the load.

[illegible] seen that the greatest B.M. that will occur is represented by the positive area GVE. This area represents the B.M. when the load is at the centre of the [illegible]

Hence

The maximum possible B.M. for a single moving load occurs directly under the load when it is at the centre of the beam.

CHAPTER XI

PULLEY SYSTEMS.

It is the intention, in the following discussion, to find out how one or more pulley blocks may be most advantageously arranged in order to lift or move a given weight.

In most of the problems the ropes of the system are assumed to be parallel to one another. This, of course, is not quite true in practice, but the error arising from such an assumption is so slight that it may be neglected, together with the errors that arise from assuming that the sheaves have frictionless bearings and that their weights are negligible.

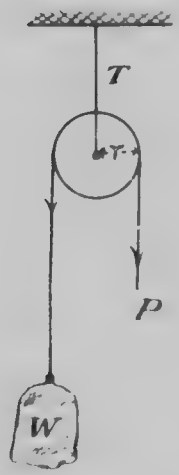


Fig. 47.

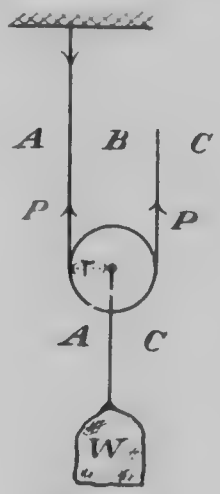


Fig. 48.

Mechanical Advantage.

The mechanical advantage of a system of pulleys [illegible] the weight lifted to the force exerted on the [illegible] of the rope.

[illegible] advantage is sometimes referred to as the **efficiency of the system**.

[illegible]

the weight W in equilibrium. Any greater force than P will lift W.

Consider the pulley as a rigid body. It is acted upon by three forces, P, W, and T, in equilibrium.

Take moments about the centre of rotation of the sheave.

$$\Sigma M = M_P + M_W + M_T = 0$$
$$P.r - W.r = 0 \quad 0.$$
$$P = W.$$

i.e., with such an arrangement of the block the force exerted at the free end of the rope to keep the system in equilibrium must be equal to the weight

$$\text{Mechanical Advantage} = \frac{W}{P} = 1.$$

II. Let the block be attached to the weight and one end of the rope made fast to some abutment as represented in Fig. 48.

Consider the pulley as a rigid body. It is acted upon by the three forces AB, BC, and CA (using Bow's Notation) in equilibrium.

Take moments about the axis of rotation as in I.

$$\Sigma M = M_{AB} + M_{BC} + M_{CA} = 0$$
$$AB.r - BC.r = 0 \quad 0$$
$$AB = BC.$$

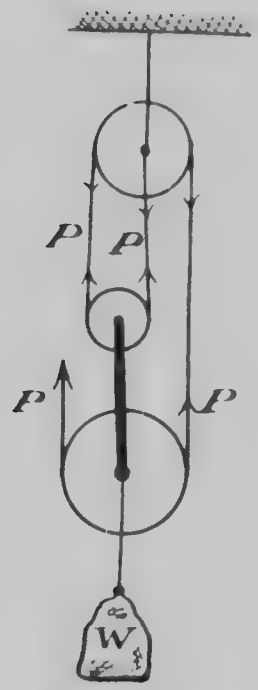
If a force of P pounds be applied at the free end of the rope, then BC = P = AB.

To find the magnitude of P —

$$\Sigma Y = Y_{AB} + Y_{BC} + Y_{CA} = 0$$
$$P + P + W = 0$$
$$P = \frac{1}{2} W$$

$$\text{Mechanical Advantage} = \frac{W}{P} = \frac{W}{W}$$

III. [illegible]



No. 19.

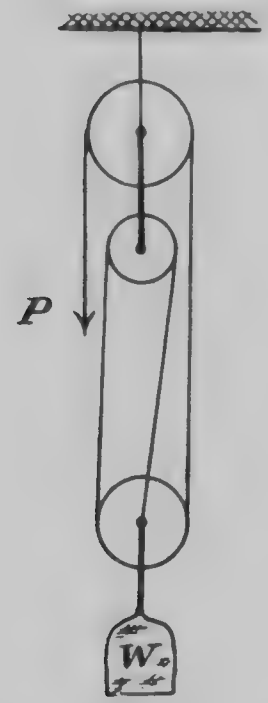


Fig. 50.

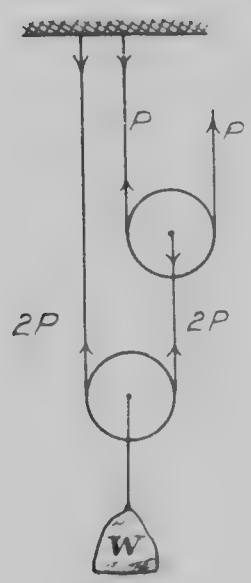


Fig. 51.

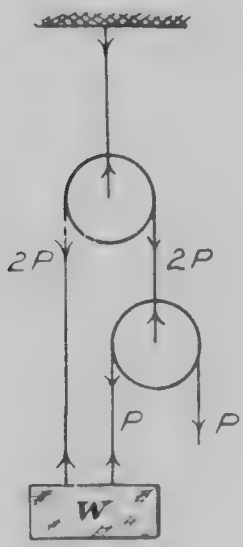


Fig. 52.

The Weston Differential Pulley

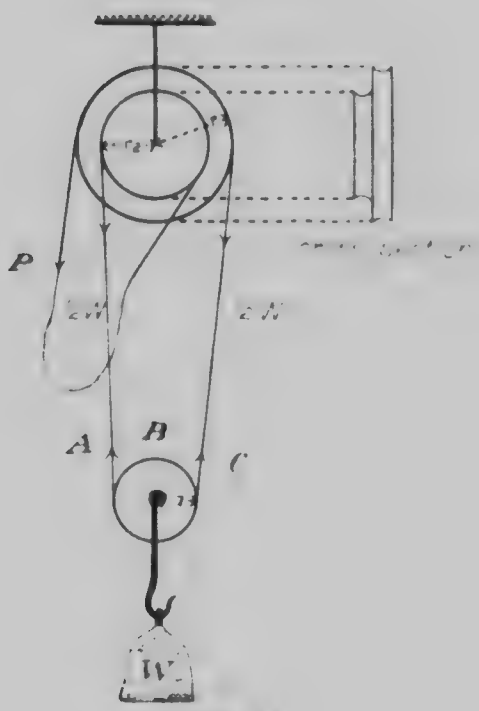


Fig. 53

[illegible]

$$\Sigma M = P \cdot \frac{W}{2} \cdot r - \frac{W}{2} \cdot r = 0$$

$$P = \frac{W}{2} \cdot \frac{r_1 - r_2}{r_1}$$

$$\text{Mechanical Advantage} = \frac{W}{P} = \frac{2r_1}{r_1 - r_2}.$$

The Mechanical Advantage of such a system is then equal to twice the radius of the large block divided by the difference between the radii of the large and small block.

To Design a Differential Pulley.

Design a differential pulley to lift a ton weight by applying ten pounds force to the free end of the system.

$$\text{Mechanical Advantage} = \frac{W}{P} = \frac{2000}{10} = \frac{2r_1}{r_1 - r_2}.$$

$$\text{Simplifying:} \quad \frac{2000}{10} = \frac{2r_1}{r_1 - r_2}.$$

$$\frac{r_1}{r_2} = \frac{200}{198}$$

so that any differential pulley whose upper blocks have their radii in the ratio of 200:198 will satisfy the demands of the problem.

What would the Mechanical Advantage be if the radii of the two upper blocks of a differential pulley were equal?

Answer: Infinite. The key to this seeming paradox is that, although the advantage is infinite, it would take an infinite time to accomplish anything with such a pulley.

Important.

In the preceding discussion of pulley systems, it is seen that it is possible to lift a large weight by the application of very little force. The student must not

fall into the trap of thinking that by doing comparatively little **work** at one end of the system that a large amount of **work** can be done at the other end. The work done at both ends of the system is exactly the same in theory. Practically, there will be a mechanical loss due to friction, etc., between the free end and the weight being lifted.

CHAPTER XII

FRICTION.

[illegible] **Friction** [illegible] point of motion. The name **Limiting Friction** is used to denote this maximum value.

There are two kinds of friction, **Sliding** and **Rolling**. [illegible]

Friction always acts in such a way as to oppose motion.

The following cases should be examined and the [illegible]

1. Ladder resting [illegible]
2. Iron wheel rolling on an elastic [illegible]
3. Rubber-tired wheel rolling on an iron plane.
4. Rope with a weight on each end, one greater than the other, stretched over a fixed pulley. (a.) Rope at rest on the point of moving. (b.) Rope moving.
5. Rod resting on a table pulled at one end perpendicularly to its length.
6. Ring laid on a rough, horizontal table and pulled at any point tangential to ring so that ring is at rest on point of motion.

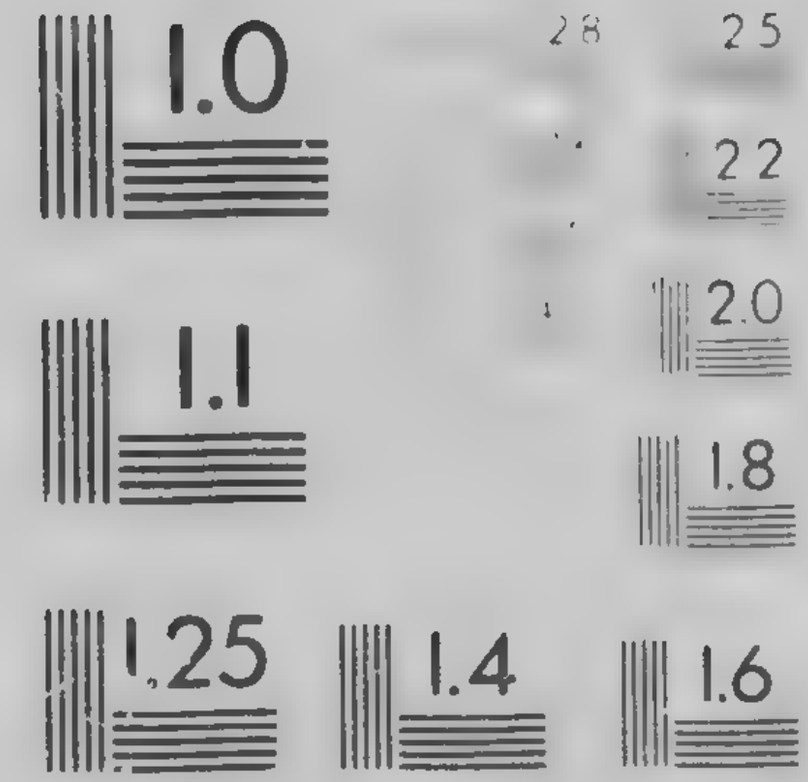
The laws of rolling friction have not been fully investigated, and will not be discussed here. Those of sliding friction are more fully understood, being based however, on experiment, and being true only within certain limits.

The chief laws are:

1. When motion is just about to take place, there is an approximately constant ratio between the friction and the **normal pressure** between the two bodies, which is called the Coefficient of Statical Friction.

This ratio is usually a small fraction, and tables are constructed empirically, giving values for different bodies.

MICROCOPY RESOLUTION TEST CHART



[illegible] takes place, [illegible] coefficient [illegible] called the Co[illegible] of D[illegible] Friction. This statement is [illegible] [illegible] velocities.

[illegible] Friction depends upon the [illegible] and [illegible] on their temperature.

Let Fig. 54 represent a body in equilibrium resting on an inclined plane, its weight W acting as indicated through the centre of gravity. There is also acting on the body the reaction, N, of the plane perpendicular to

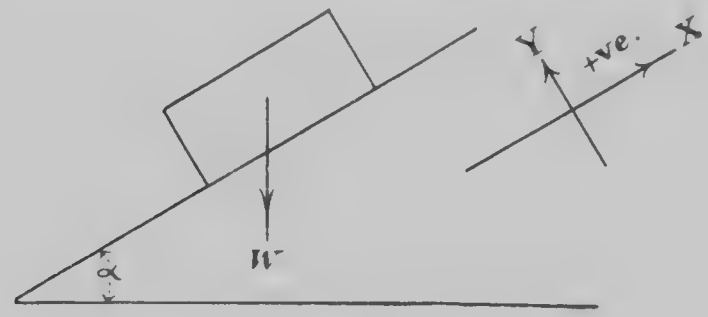


Fig. 54.

the surface of contact and the friction F. These forces acting on the body are in equilibrium. Therefore, $\Sigma X = 0$; $\Sigma Y = 0$; and $\Sigma M = 0$.

Taking the axis of X parallel to surface of plane as indicated

$$\Sigma X = -W \cos (90 - \alpha) + F = 0.$$

$$F = W \sin \alpha.$$

$$\text{or } W = \frac{F}{\sin \alpha} \quad \ldots\ldots\ldots (1.)$$

$$\Sigma Y = W \sin (90 - \alpha) + N = 0.$$

$$N = W \cos \alpha \ldots\ldots (2.)$$

Substitute value of W from (1.) into (2.)

$$N = \frac{F}{\sin \alpha} \cos \alpha$$

$$\text{or } \frac{F}{N} = \tan \alpha \ldots\ldots\ldots (3.)$$

Now, imagine the plane as being gradually tipped up so that the angle of inclination α becomes greater and greater. A position will be eventually reached beyond which the plane cannot be raised without causing the body to slip. The plane is then said to be inclined at the **"angle of repose."**

Definition: —

If a body rest on a plane surface, the Angle of Repose is that angle beyond which the plane cannot be inclined without the body moving down the plane.

When the plane reaches the angle of repose from (3).

$$\frac{F}{N} = \text{tan. angle of repose}$$

$\frac{F}{N}$, in this particular instance, is known as the **coefficient of friction,** usually designated by the Greek letter μ.

Coefficient of Friction μ tangent of angle of repose.

From the previous discussion it is seen that if a body rest on a plane surface which is inclined at an angle less than the angle of repose (sometimes called the angle of friction), the body will not slip down the plane. If now, some external force be applied to the body so as to cause the body to be **just on the point of moving** either up or down the plane, the resistance offered by friction can in every such case be experimentally shown to be of a magnitude given by

$$F = N.\mu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.)$$

Where F = frictional resistance.
N pressure normal to the plane.
μ coefficient of friction for plane and body.

Problems.

A body weighing 100 pounds rests on a plane surface which is inclined to the horizontal at an angle of 30°, the angle of repose for the plane and body being 45°. Find what force acting parallel to the plane will

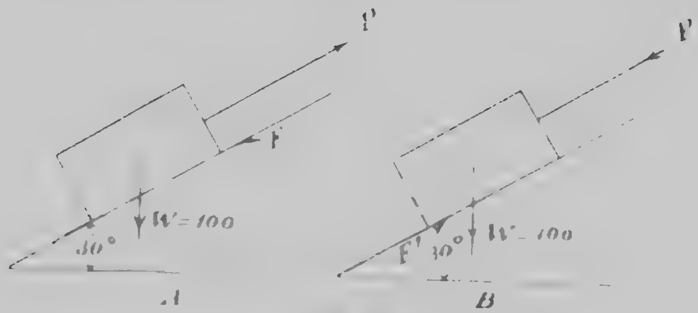


Fig. 55.

[illegible]

Therefore, F = [illegible]

[illegible]

Referring to Fig. 55 B [illegible] P represents the required force, [illegible] F, which in this case acts [illegible] against the tendency to motion, the friction [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

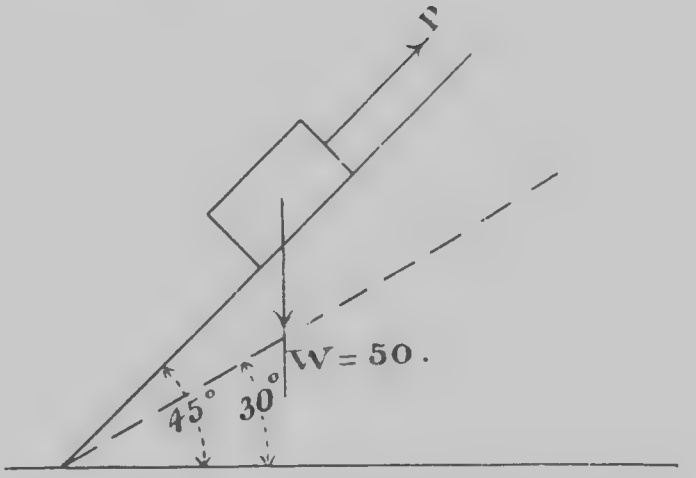


Fig. 56.

Fig. 56 represents the condition of [illegible]

$$\Sigma X = X_W + X_P + X_F = 0$$

$$50 \cos 45 - P + F = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7.)$$

$$F = N.\mu.$$

$$50 \cos 45 \cdot \frac{1}{\sqrt{3}} = 50 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}}.$$

Substituting this value of F into (7.),

$$-50\frac{1}{\sqrt{2}} - P - \frac{50}{\sqrt{2}}\cdot\frac{1}{\sqrt{3}} = 0$$

$$P = \frac{50}{\sqrt{2}}\left(1 + \frac{1}{\sqrt{3}}\right).$$

CHAPTER XIII

WIND PRESSURE.

Pressure on Surfaces Normal to Direction of Motion.

one second

PROBLEMS IN APPLIED STATICS.

CHAPTER [illegible]

MISCELLANEOUS PROBLEMS.

If the Vector Polygon be drawn for a set of forces which are in equilibrium, the polygon must close.

[illegible]

Fig. 58.

[illegible]

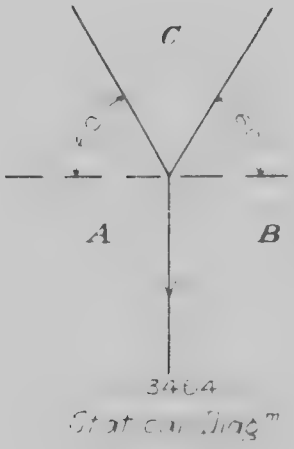


Fig. 59.

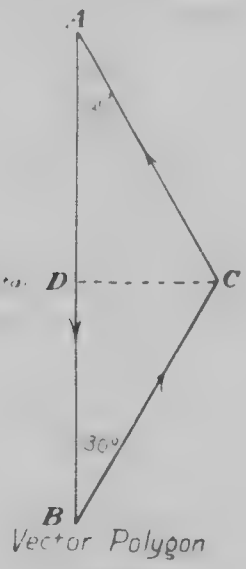


Fig. 60.

[illegible] BC and C[illegible]

[illegible] BC and CA [illegible] Diagram, it is seen that both BC and CA [illegible]

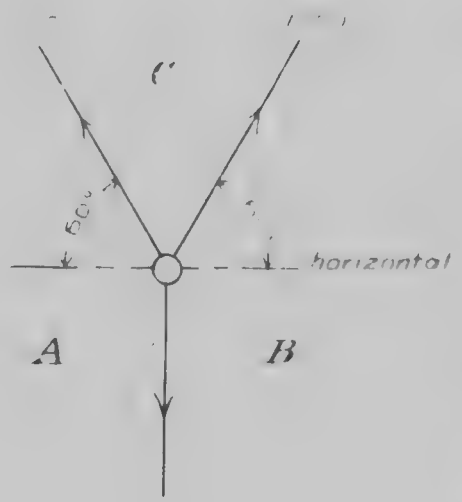


Fig. 61.

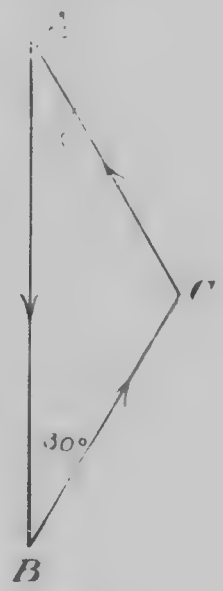


Fig. 62.

In order to find the magnitude of BC and CA, [illegible] CD perpendicular on AB

[illegible] DB

[illegible] AB [illegible]

[illegible] DB

DB [illegible]

[illegible]

[illegible]

[illegible]

CA

[illegible]

[illegible]

in tension [illegible]

[illegible]

If the chain (Fig. 58) can safely stand tension to the extent of 1,500 pounds, what is the maximum weight that may be placed on the hook?

Let the chain be stressed [illegible]

[illegible]

Fig. 63.

[illegible]

[illegible]

[illegible] to 1,000 pounds applied at the outside [illegible] **It is required to find the stress in the members of the truss.**

Consider the forces acting at the point ABC [illegible] Here is a known force BC of 1,000 pounds [illegible] downward, and two [illegible]

Fig. 64. **Fig. 65.** **Fig. 66.**

CA and AB, whose lines [illegible] Statical Diagram (Fig. 64) represents this condi[illegible]

[illegible] point B (Fig. 65) construct BC [illegible] force BC.

Now, **since the forces under consideration are in equilibrium, their vector polygon must close.** [illegible] C and B (Fig. 65) draw lines to [illegible] CA and AB, [illegible] CA and AB [illegible] forces CA and AB, [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

If a set of forces is in equilibrium, the algebraic sum of the moments of the forces about any point is equal to zero.

Fig. 80 represents [illegible] a system of levers in a weighing scale. Of [illegible]

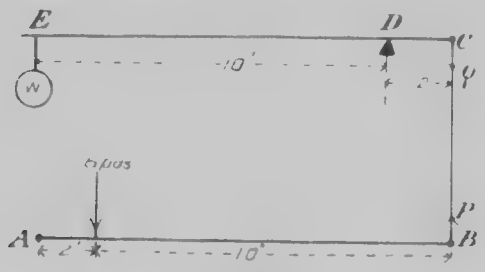


Fig. 80.

[illegible] in such a figure, but nevertheless [illegible]

AB and EC are two levers, the former being hinged [illegible] A and the latter resting on a knife edge D. The [illegible] BC is connected to these two levers at B and C [illegible]

A force of six pounds is exerted on the lever AB at a distance of two inches from the hinge A. What is the magnitude of the weight W placed on the upper lever at E such that the whole system is in equilibrium?

Consider the lever AB. It is acted upon by the [illegible] force of six pounds, [illegible]

vertical force P at B, and the reaction at the hinge A. These forces are in equilibrium; therefore, if moments be taken about any point, $\Sigma M = 0$.

Take moments about the point A.

$\Sigma M = (\text{reaction at } A \times \text{zero}) \times 6 \times 2 \times P \times 12 = 0$ [illegible]

Since we do not know whether P [illegible] downward, [illegible] of P is [illegible] positive.

$$P \times 12 = [illegible]$$
$$P = [illegible]$$

This negative result shows that the moment of P about A is negative; i.e., P acts upward at B as indicated by the arrowhead, and is of magnitude [illegible] pound.

The rod BC exerts a force away from B. It [illegible], therefore, exert an equal and opposite force Q [illegible] other extremity C; i.e., at C there is a vertical force of one pound acting downward as indicated by the [illegible] head.

Consider the lever EC. It is acted upon by three forces in equilibrium, viz., the force W acting at E, the reaction of the knife edge at D, and the known vertical force acting at C. For this set of forces, ΣM about any point equals zero.

Take moments about D:

$\Sigma M = Q \times 2 \times (\text{reaction at } D \times \text{zero}) \times W \times 10 = 0$ [illegible]

Putting in the known value of Q [illegible], we get:

$$W \times 10 = [illegible]$$
$$W = [illegible]$$

[illegible], W is a weight of [illegible] pound, [illegible] a manner as to give a negative moment about D. W, therefore, [illegible] downward.

[illegible] the moments of P and W in equations [illegible] his experience would [illegible] quantities [illegible] convention [illegible] be [illegible] that P [illegible] W [illegible] giving [illegible] [illegible] generally [illegible] and [illegible] [illegible]

If a force P of 100 pounds be exerted at the joint of the upper arm as indicated, what force S must be exerted at the joint of the other arm in order to preserve equilibrium?

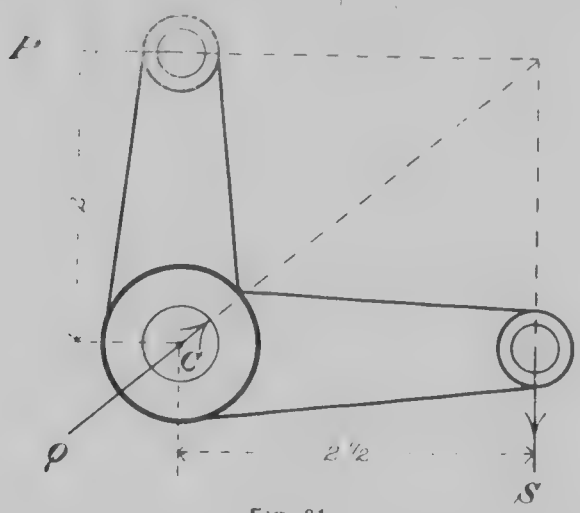


Fig. 81.

the crank

equal but opposite to S

To Find the Reaction Q:

CHAPTER XV

SIMULTANEOUS USE OF EQUATIONS.

$\Sigma X = 0$ and $\Sigma Y = 0$.

In the work previous to this, it has always been possible to determine the unknown forces of a set, one at a time, by the independent use of either the equations $\Sigma X = 0$ or $\Sigma Y = 0$. In the case that will be taken up next, and in many other cases, it is only possible to arrive at results by the simultaneous use of the two equations $\Sigma X = 0$ and $\Sigma Y = 0$, and in such cases certain precautions must be observed.

The roof truss (Fig. 95) is known as a German Truss. This form of structure is obsolete as far as prac-

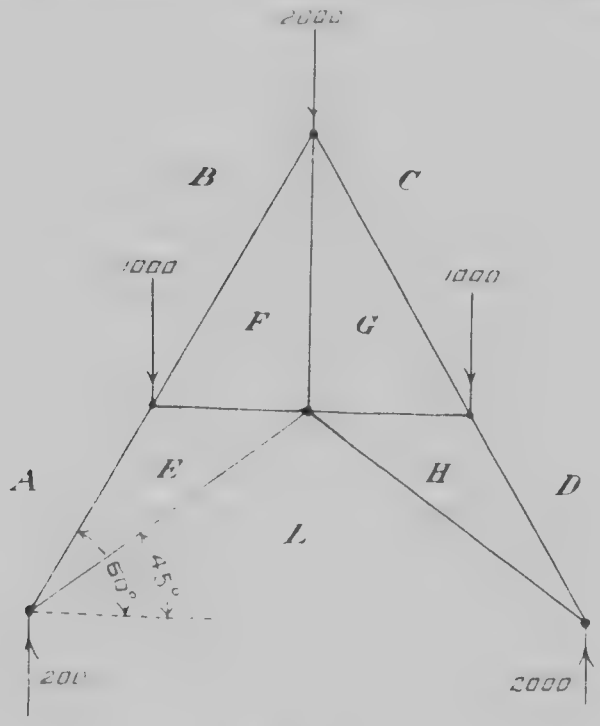


Fig. 95.

[illegible] the purpose of illustrating the point under dis-

Considering the forces acting on the truss and applying the equation $\Sigma M = 0$, taking moments about a point in the line of action of either abutment reaction, it is seen that the abutment reactions are both equal to 2000 pounds.

Consider the forces acting at the point LHD (Stress Diagram, Fig. 97). Since these forces are in equilibrium, $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$.

$$\Sigma X = X_{LH} + X_{HD} + X_{DL} = 0.$$

LH and HD being both, as shown, tension, as are their X's positive.

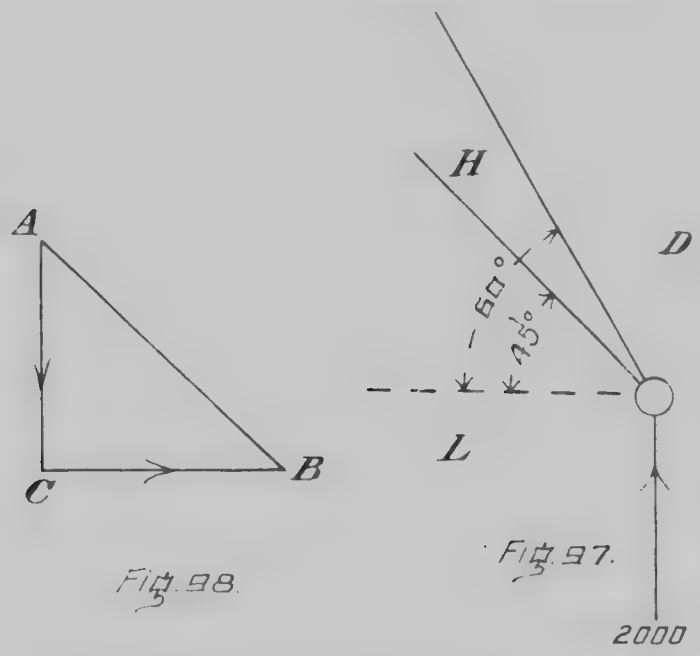


Fig. 98.

Fig. 97.

$$LH \cos 45 + HD \cos 60 = 0 = 0$$

$$LH \frac{1}{\sqrt{2}} + HD \frac{1}{2} = 0$$

$$\sqrt{2}\, HD$$

$$LH$$

$$1 \quad 2.$$

$$HD$$

$$\sqrt{2}$$

The assumptions as to the signs of the Y's of the unknown forces must be in accordance with the assumptions as to the signs of the X's of the unknown forces in the first place.

[illegible]

HD

[illegible]

HD

[illegible]

[illegible] X_{HD} and [illegible]

The Y_{HD} was assumed negative and X_{HD} positive [illegible] to be correct; the force HD [illegible] the point; i.e., the member HD [illegible]

[illegible]

[illegible] of HD [illegible]

HD

LH [illegible]

[illegible]

LH

[illegible]

It is evident from the negative [illegible] assumed signs of X_{LH} and Y_{LH} [illegible] X_{LH} is negative and Y_{LH} positive. The force LH, [illegible]

[illegible]

LH [illegible]

[illegible]

[illegible] been used first, although the [illegible] The [illegible] preceding discussion is that, **if the equations have to be used simultaneously, the assumptions as to signs in the second equation must be in accordance with the given lines of action of the unknown forces and the assumed signs in the first equation.**

[illegible]

Consider the forces acting at the point LEA.

CHAPTER XVI

DETERMINATION OF STRESS IN BRIDGE AND ROOF TRUSS MEMBERS.

[illegible] of the stress in the members of a framed structure, **the joints of the truss must be considered as frictionless circular pin joints, perfectly fitted so as to allow of no slack motion,** the [illegible] the theory is based. So, then, whenever [illegible] encounters the [illegible] at a point in a truss, he must be [illegible] really considering **the forces which act on the pin of the joint,** for [illegible] is being considered, there [illegible] acted upon. In [illegible] the members are the bodies acting on it. [illegible] pointed out, however, **that because of this construction,** the lines of action of the forces act[illegible] seen at the centre of the pin, and [illegible] referred to in the statement: "To consider the [illegible] acting at a point." With the exception of the case of three forces in equilibrium, which, of course, **must** [illegible] directions intersecting [illegible] be thought that the forces acting at a joint in [illegible] must act through the centre of the pin in order to [illegible] equilibrium. **They merely do so,** as pointed out before **because of the construction of the joint.**

Find the stress in the members of a cantilever such as indicated in Fig. 67, the panels of which are equilateral triangles.

Consider the point BCA. [illegible]

[illegible] **important** [illegible] **at the point being considered** [illegible] **the lines of action of forces acting on the pin**

FIG. 17.

of AP and PB in Fig. 70, it is seen that AP acts away from [illegible] and PB against it.

[illegible] AP is, therefore, in tension [illegible] pounds, and the member PB in compression [illegible] pounds.

Consider the point PACDE. Acting at this point are [illegible] the load and [illegible] forces exerted by the tension members PA and AC and two unknown forces as indicated in the Statical Diagram (Fig. 72).

PA, AC, and CD (Fig. 73) represent the known forces PA, AC, and CD. From D and P, respectively, draw [illegible] parallel to the directions of DE and EP (Fig. 7[illegible]), intersecting at E. DE and EP (Fig. 73) represent, respectively, the unknown forces DE and EP, acting [illegible] by the continuously pointed sense marks. In order to avoid confusion, another diagram (Fig. 7[illegible]) is [illegible], showing how to arrive [illegible] of DE and EP. From this diagram [illegible] DE and EP [illegible] pounds [illegible].

The force DE evidently acts away from the point and EP [illegible] the point. The member DE is, therefore, in tension [illegible] pounds, and the member EP in compression [illegible] pounds.

Find the stress in the remaining members LG, GB, EF, and FG.

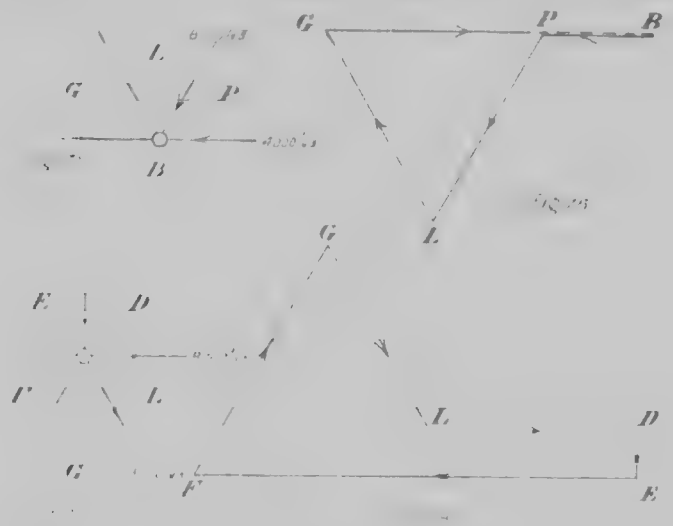
[illegible] BPEG, [illegible] GEDEF.

[illegible] BPEG, [illegible]

To Construct a Stress Diagram.

In the last problem a new Vector Polygon was constructed for each set of forces being considered. If these diagrams are drawn to a suitable working scale, they spread over a considerable area, and also, in transferring magnitudes to a new line, there is the liability of an error which may or may not be cumulative in its effect throughout the problem. It is possible, however, to condense these diagrams and at the same time restrict the liability to error in magnitudes by drawing what the writer chooses to call a **Continuous Vector Diagram**, or, as it is often referred to, a **Stress Diagram**.

It may be pointed out that it will help to simplify the problem if the reader will construct, on a loose sheet of paper, the Vector Polygons (Figs. 69, 71, 73, 79, and



[illegible] the same scale of forces in each case. [illegible] **the same scale of forces will be used throughout the following problem:**

[illegible]

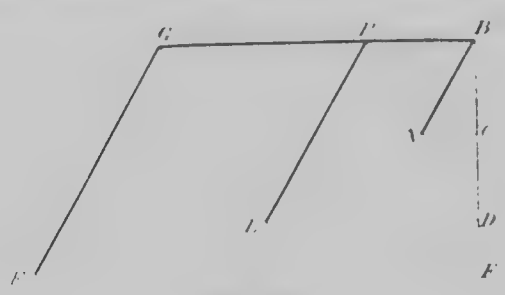


Fig. 79.

[illegible] the point GLDEF [illegible] diagram when drawn should read, GL, LD, DE, EF, and FG.

Doubtless, the reader will see that the construction of this Stress Diagram is merely a process of piecing together the Vector Polygons for the various points of the truss.

It should be noticed that the lines of the Stress Diagram, since they have no sense marks placed on them, really represent the magnitude of the stress in the various members; e.g., the line LG represents the stress in the member LG.

In the two preceding problems the determination of the stress in the various members of the cantilever truss (Fig. 67) was arrived at by graphical means, although a little trigonometry was occasionally made use of. It is possible, however, to arrive at these quantities analytically as indicated in the following discussion. In order to simplify matters, the reader is advised to draw the Statical Diagrams (Figs. 68, 70, 72, 75, and 77) on a loose sheet of paper so that they may be readily referred to.

The following facts must be kept in mind throughout the problem:

The **magnitude** of the X of any force = (magnitude of force) × (cosine of angle of inclination of force to the horizontal).

The **magnitude** of the Y of any force = (magnitude of force) × (sine of angle of inclination of force to the horizontal).

Also, [illegible] $\Sigma X = 0$, and $\Sigma Y = 0$.

Analytical Determination of the Stress in the Members of the Cantilever (Fig. 67).

[illegible]

$Y_{BC} = BC \sin \theta = BC = 2{,}000$

$Y_{CA} = CA \sin \theta = \frac{\quad}{\sqrt{3}}$

$Y_{AB} = AB \sin \theta = AB \frac{\quad}{2} \ldots\ldots\ldots\ldots 2$

Substituting these values in equation [illegible]

$\sqrt{3}$

$\Sigma Y = \ldots \sin \theta = AB \frac{\quad}{2} \ldots$

$2 \quad 2{,}000$

$AB = \ldots$

$\sqrt{3} \quad \sqrt{3}$

If, now, this [illegible] of AB, which is a positive quantity, be substituted for AB in equation [illegible], it is seen that the assumed positive sign of the Y of AB remains unchanged. The assumed sign of the Y of AB is, therefore, correct, i.e., the Y of AB is positive.

If Y_{AB} is positive, i.e., [illegible] upward, AB is found to be [illegible] against the pin. The [illegible] AB is,

$4{,}000$

therefore, in compression [illegible] pounds.

$\sqrt{3}$

For the same point:

$\Sigma X = X_{BC} - X_{CA} - X_{AB} = \ldots\ldots\ldots 3$

$X_{BC} = BC \cos \theta = \ldots$

$X_{CA} = CA \cos \theta = CA$ [illegible] assumed positive since CA is unknown. 4

$4{,}000 \quad 1 \quad 2{,}000$

$X_{AB} = AB \cos \theta = AB \ldots$

$\sqrt{3} \quad 2 \quad \sqrt{3}$

Substituting these values in [illegible]

$2{,}000$

$\Sigma X = \ldots CA \ldots$

$\sqrt{3}$

$2{,}000$

CA

$\sqrt{3}$

[illegible] CA [illegible] $\sqrt{3}$ [illegible] CA is [illegible] $2{,}000 \sqrt{3}$ [illegible]

The forces acting at the point BAP, as shown in the Statical Diagram (Fig. 70), are in equilibrium

$$\Sigma Y = Y_{BA} + Y_{AP} + Y_{PB} = 0$$

$$BA \sin 60^\circ + AP \sin 60^\circ + PB \sin 0^\circ = 0$$

$$\frac{4{,}000 \times \sqrt{3}}{2} + AP \frac{\sqrt{3}}{2} + 0 = 0$$

$$AP = -\frac{4{,}000}{\sqrt{3}}$$

From the positive sign of this result the Y_{AP} is seen to be positive; i.e., the force AP acts away from the pin. The member AP is, therefore, in tension $\frac{4{,}000}{\sqrt{3}}$ pounds.

For the same point,

$$\Sigma X = X_{BA} + X_{AP} + X_{PB} = 0$$

$$BA \cos 60^\circ + PB \cos 60^\circ + PB \cos 0^\circ = 0$$

$$\frac{4{,}000}{\sqrt{3}} \times \frac{1}{2} + \frac{4{,}000}{\sqrt{3}} \times \frac{1}{2} + PB = 0$$

$$PB = -\frac{4{,}000}{\sqrt{3}}$$

The positive sign of the result shows that the X_{PB} is positive. PB must, therefore, act against the pin, placing the member PB in compression $\frac{4{,}000}{\sqrt{3}}$ pounds.

The forces acting at the point PACDL are in equilibrium, and act as represented in the Statical Diagram (Fig. 72).

$$\Sigma Y = Y_{PA} + Y_{AC} + Y_{CD} + Y_{DL} + Y_{LP} = 0$$

$$PA \sin 60^\circ + AC \sin 0^\circ + CD \sin 60^\circ + DL \sin 0^\circ + LP \sin 60^\circ = 0$$

$$\frac{4{,}000 \times \sqrt{3}}{2} + 0 + \frac{4{,}000}{\sqrt{3}} \times \frac{\sqrt{3}}{2} + 0 + LP \frac{\sqrt{3}}{2} = 0$$

$$LP = -\frac{8{,}000}{\sqrt{3}}$$

[illegible]

[illegible] shows that the Y_{LP} [illegible] In accordance with this, LP [illegible] the pin, placing the member LP in [illegible] $\sqrt{3}$ pounds.

[illegible]

$$\Sigma X = X_{BP} + X_{PC} + X_{CD} + X_{DL} + X_{LG} = 0$$

$$PA \cdot \cos\theta + AC \cdot \cos\theta + CD \cdot \cos\theta$$

$$DL \cdot \cos\theta + LP \cdot \cos\theta = 0$$

$$\frac{1{,}000}{\sqrt{3}} \cdot \frac{1}{2} + \frac{2{,}000}{\sqrt{3}} \cdot \frac{1}{2} + \frac{8{,}000}{\sqrt{3}} \cdot \frac{1}{2}$$

$$0 + DL \cdot \frac{1}{2} = 0$$

$$\sqrt{3} \cdot \frac{1}{2} + \sqrt{3} \cdot \frac{1}{2} + \sqrt{3} \cdot \frac{1}{2}$$

$$\frac{8{,}000}{\sqrt{3}}$$

$$DL$$

The negative sign of the result shows that the X_{DL} (since the X equation is being used) is negative. DL, therefore, acts away from the pin; i.e., the member DL is in tension $\frac{8{,}000}{\sqrt{3}}$ pounds.

The reader is advised to try and determine the stresses in the remaining members before referring to the solution as herein given.

For the forces acting at the point BPLG (Statical Diagram, Fig. 75):

$$\Sigma Y = Y_{BP} + Y_{PL} + Y_{LG} + Y_{GB} = 0$$

$$\frac{8{,}000}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot LG + 0 = 0$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{8{,}000}{\sqrt{3}}$$

$$LG$$

From this positive result it is seen that Y_{LG} is positive. LG, therefore, acts away from the point; i.e., the member LG is in tension $\frac{8{,}000}{\sqrt{3}}$ pounds.

[illegible]

$$\Sigma X = X_{BP} + X_{PL} + X_{LG} + X_{GB} = 0$$

$$\frac{1{,}000}{\sqrt{3}} \cdot \frac{1}{2} + \frac{8{,}000}{\sqrt{3}} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot GB$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$$

$$GB$$

$$\sqrt{3}$$

The X_{GB} is evidently positive. GB, [illegible] acts against the pin. The member GB [illegible] compression $\frac{12,000}{\sqrt{3}}$ pounds.

Considering the point GLDEF (Statical Diagram, Fig. 77)

$$\Sigma Y = Y_{GL} + Y_{LD} + Y_{DE} + Y_{EF} + Y_{FG} = 0$$

$$\frac{8,000}{\sqrt{3}} \cdot \frac{1}{2} + 0 + 1,000 + 0 + FG \cdot \frac{1}{2} = 0$$

$$FG = \frac{10,000}{\sqrt{3}}$$

The Y_{FG} being positive (from the positive result) FG must act against the pin; i.e., the member FG is in compression $\frac{10,000}{\sqrt{3}}$ pounds.

For the same point:

$$\Sigma X = X_{GL} + X_{LD} + X_{DE} + X_{EF} + X_{FG} = 0$$

$$\frac{8,000}{\sqrt{3}} \cdot \frac{1}{2} + \frac{8,000}{\sqrt{3}} + 0 + EF + \frac{10,000}{\sqrt{3}} \cdot \frac{1}{2} = 0$$

$$EF = \frac{17,000}{\sqrt{3}}$$

The negative result shows that X_{EF} is negative. In accordance with this, EF acts away from the point. The member EF is, therefore, in tension $\frac{17,000}{\sqrt{3}}$ pounds.

It is seen that these results check those found by graphical means. It must be recognized, however, that the so-called graphical solutions taken up so far in these pages have not been [illegible]. In [illegible] constructing the diagrams and finding the magnitudes of the forces by scaling the lengths of the lines composing the Vector Polygon, trigonometrical means have been used to get at these quantities. The solutions have, therefore, been partly analytical. The point of the [illegible] [illegible] employed there would have been found an error in the [illegible] [illegible] of the forces [illegible] to the [illegible]

[illegible] that [illegible] so small that [illegible] it does not amount to anything. Analytical results are, of course, perfectly accurate.

King Post Truss.

Fig. 82 represents a simple form of [illegible] bridge truss, known as a King Post Truss. It must be clearly understood, however, that the ordinary forms of roof bracing, consisting of the rafters and scantling tie-rods, which resemble the above truss in outline, do not really present the same problem as will be herein discussed. Usually, the rafters have the [illegible] sheeting, shingles, etc., lying directly on them over their entire length. In this case, the rafters are inclined [illegible] distributed load, and the stress in them will not be simple Compression, but a combination of both Tension and Compression due to bending. If, however, stringers or purlins be laid from truss to truss on the roofing system so as to lie at the joints of the trusses, or nearly so, and if the rafters and roofing be built on these stringers, the load will then be transferred to the truss mere [illegible] the joints, and the case [illegible] then be worked out [illegible] methods taken up so far. Fig. 82 A

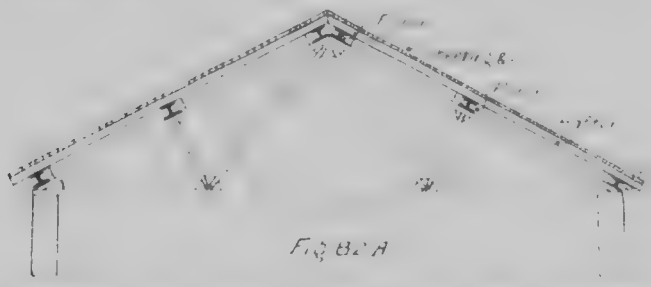


Fig. 82 A

[illegible] of roof truss, the method of [illegible] and building on them.

There is also a form of bridge truss very commonly met with on country roads which is sometimes [illegible] King Post Truss. The structure referred to [illegible] a heavy timber beam [illegible] between the [illegible] the trussing above merely serves to stiffen this beam. In this case, the beam, which seemingly corresponds to the [illegible] of the ordinary King Post Truss, is [illegible] Tension, but [illegible] Compression existing in it, and these stresses [illegible] elementary methods.

Required to find the stress in the members of the truss (Fig. 82).

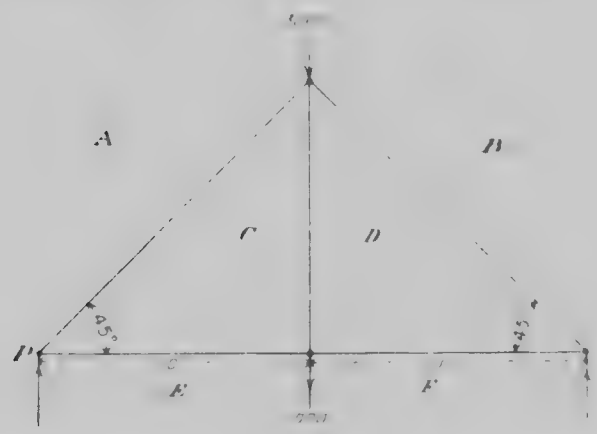


Fig. 82.

Analytical Solution: -

The whole truss is a body acted upon by a set of outside forces. These forces, the two loads AB and EF, and the two abutment reactions BF and EA, are in equilibrium; therefore, $\Sigma X = 0$; $\Sigma Y = 0$; $\Sigma M = 0$.

Take moments about a point in the line of action of one of the abutment reactions, say, the point P

$$\Sigma M = M_{AB} + M_{EF} + M_{BF} + M_{EA} = 0.$$

Since BF is unknown, assume M_{BF} positive.

$$3{,}000 \times 10 + 1{,}000 \times 10 + BF = 20$$

$$EA \times 0 = 0$$

$$BF = 2{,}000.$$

From the negative sign of the result, it is seen that M_{BF} about P is negative; i.e., BF acts upward.

$$\Sigma Y = Y_{AB} + Y_{EF} + Y_{BF} + Y_{EA} = 0.$$

$$3{,}000 + 1{,}000 + 2{,}000 + EA = 0.$$

$$EA = 2{,}000$$

From the positive result we infer that Y_{EA} is positive; i.e., EA acts upward.

Consider the forces acting on the [illegible] at the point EAC. The known and unknown forces are represented in the Statical Diagram (Fig. 83).

[illegible]

[illegible]

The positive [illegible] shows that [illegible] positive CL, therefore, acts away from the pin, i.e., the member CL is in tension [illegible] pounds.

Consider the point BED. The forces acting on the [illegible] indicated in the Statical Diagram, Fig. 8[illegible]:

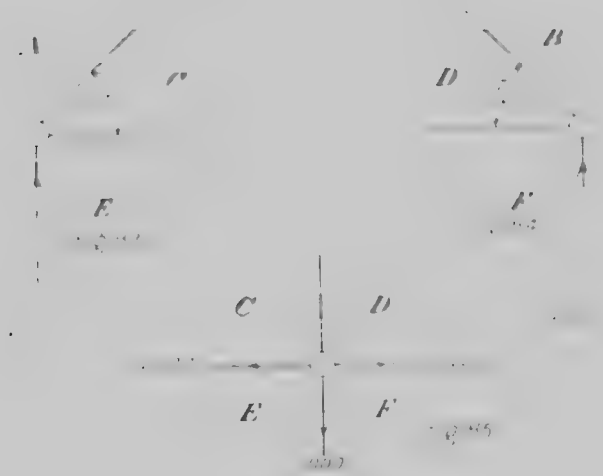
[illegible]

From the negative result, the [illegible] is seen to be negative. DB, therefore, acts against the pin, i.e., the member DB is in compression [illegible] pounds.

[illegible]

[illegible]

[illegible]



$$-Y = Y_{FE} - Y_{FD} - Y_{DC} - Y$$

[illegible] DC [illegible]

DC = 1,000.

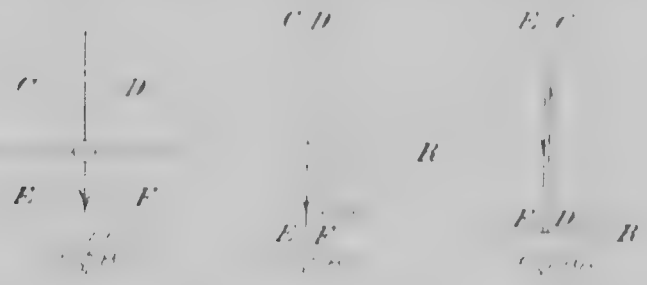
From the positive [illegible] DC, therefore, [illegible] DC [illegible]

Graphical Solution.

Consider the forces acting at the point EFDC, represented in the Statical Diagram (Fig. 86). S[illegible] [illegible] unknown forces in [illegible] impossible to construct the Vector Pol[illegible] [illegible] forces. However, **because FD and CE have the same lines of action, their resultant must act in that line.** Replace FD and CE by their resultant as indicated [illegible] R, Statical Diagram (Fig. 87). How this res[illegible] [illegible], this is of [illegible]

[illegible] Vector Pol[illegible] [illegible] Statical Diagram (Fig. 87). Draw EF (Fig. 88 [illegible] represent the known force EF (Fig. 87). Then, [illegible]

[illegible]



[illegible]

Consider the forces acting at the point ABDC, [illegible]

AB [illegible] **Whenever this notation cannot be applied to a line representing any particular force, the line will be shown dotted and the letters designating the force placed beside the line.**

[illegible]

[illegible] CA. [illegible] Bow's Notation applies to the [illegible] CA. [illegible] is, therefore, [illegible] should then read AB, DC [illegible], BD [illegible] and CA.

Having [illegible] now possible to [illegible] Vector Polygon [illegible] Bow's Notation.

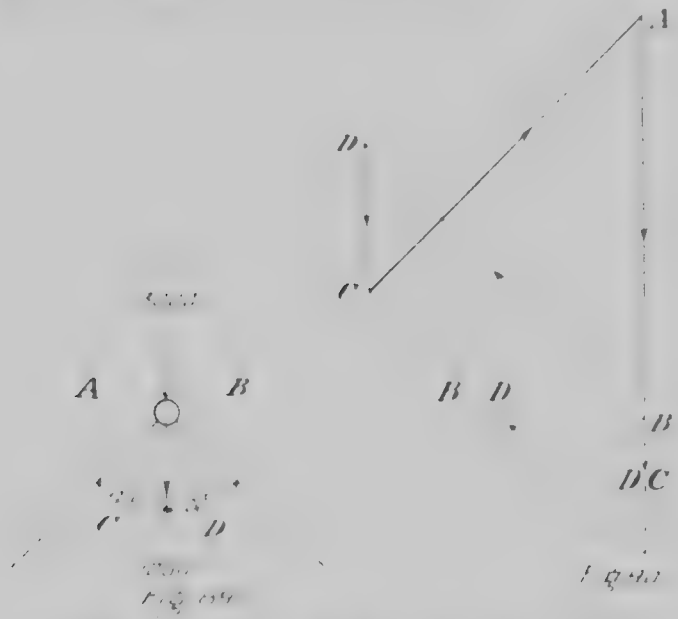


Fig. [illegible]

Fig. [illegible]

From B [illegible] BD [illegible] BD [illegible] DC [illegible] DC [illegible] AB, BD, DC [illegible] CA.

[illegible] BD [illegible] CA [illegible] BD [illegible] BD [illegible] CA [illegible]

[illegible] DE [illegible] CE [illegible] Vector Polygons [illegible]

Stress Diagram

[illegible] Fig. 82 is as follows: [illegible]

Considering the point ABDC [illegible] seen that the lines AB, DC (dotted line), BD [illegible] CA [illegible] form a [illegible]

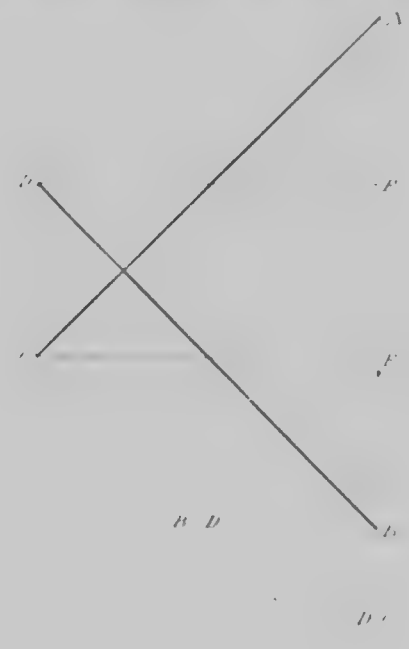


Fig. 95.

[illegible] a heavy line will be used to indicate forces acting against the point, due to members in compression; a light line indi-

cating forces acting away from the point, due to tension members; a double line being used to represent forces due to loads or abutment reactions, whether they act against or away from the point being considered.

The Queen Post Truss

Stress Diagram.

Fig. 96 is the Stress Diagram for the truss, [illegible] the same system of double [illegible] the preceding Stress D[illegible]

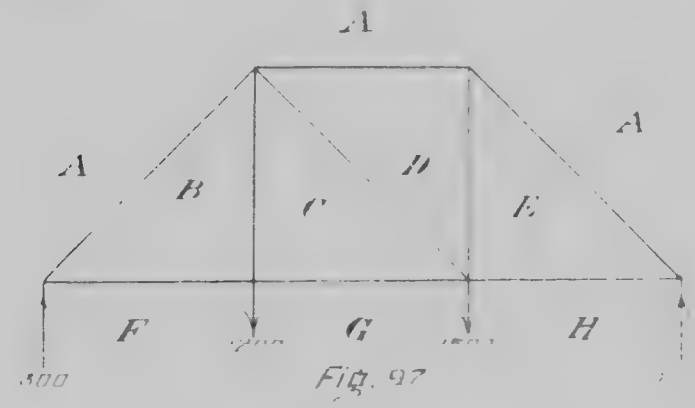


Fig. 97

The reader is advised to construct Fig. [illegible] eet of paper, and, as the stress in e[illegible] and, indicate it on this diagram. This will [illegible] clearly how the various Statical D[illegible]

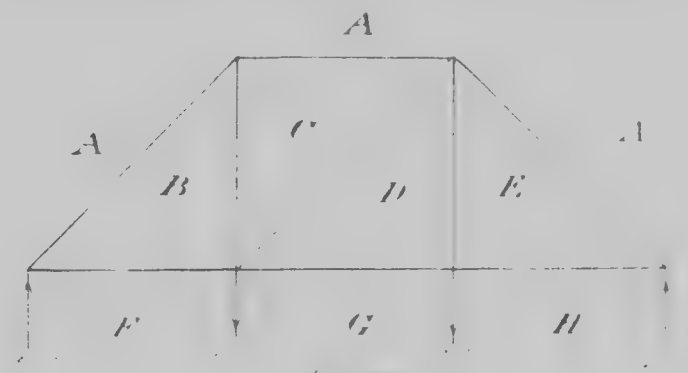


Fig. [illegible]

[illegible] when the point is first considered.)

If the following Vector P[illegible]

Consider the point AFB [illegible]

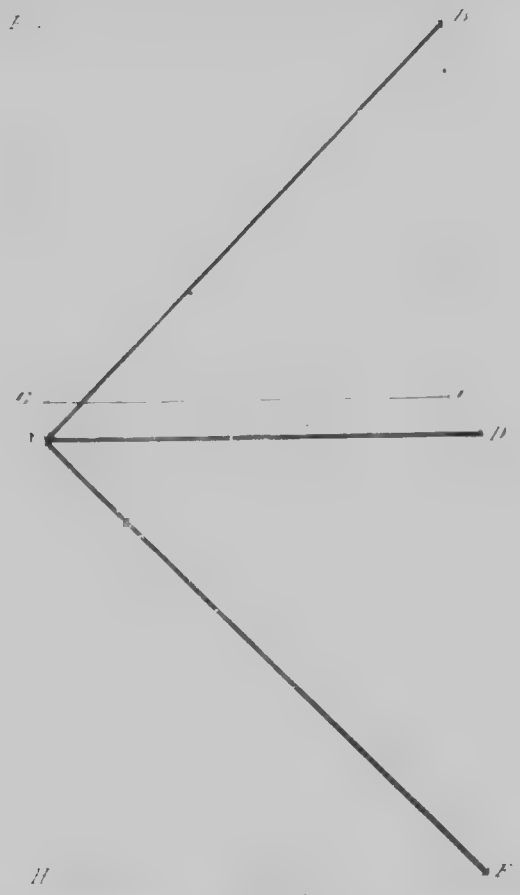


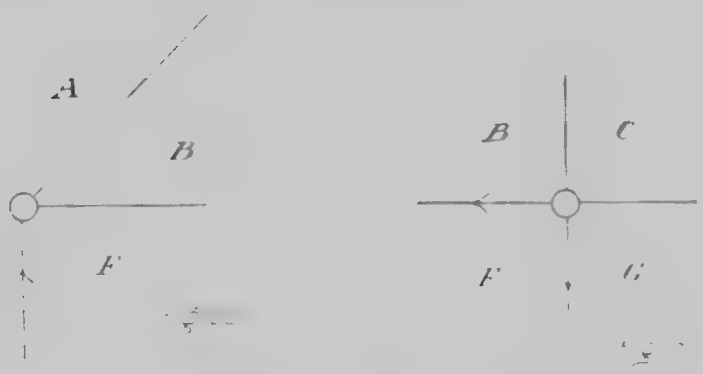
Fig. 99.

[illegible]

Consider the point BFGC (Statical Diagram, [illegible]

[illegible] ember BF (or FB). This force, is, there[illegible] [illegible] ber at the point AFB. But FB (Fig. 99) [illegible] exerted at the point AFB; therefore, [illegible] ead BF will represent the force BF [illegible] BFGC. The line FG (Fig. 99) represents the lo[illegible] FG. (The load FG is less than the reaction AF.) [illegible] lines GC and CB complete the polygon, which [illegible] read: BF, FG, GC, and CB. The members G[illegible] d CB are evidently both in tensio[illegible]

Consider the point ABCD (Statical Diagram, [illegible]



The Vector Polygon for this point reads on Fig. [illegible] AB, BC, CD, and DA, from which it is seen that th[illegible] force CD acts away from the point and the force DA [illegible]gainst the point. The member CD is, therefore, [illegible]ension and the member DA in compressio[illegible]

Consider the forces acting at the point ADE (Stat[illegible] Diagram, Fig. 103). Referring to Fig. 99, th[illegible] Vector Polygon for these forces reads: AD, DE, [illegible] EA, from which it is seen that the member DE [illegible] [illegible]ion and the member EA in compressio[illegible]

Finally, **from a consideration of the point EDCGH** (Statical Diagram, Fig. 104), the member HE [illegible] [illegible] Fig. 99, ED, DC, CG, GH [illegible] [illegible] HE [illegible] [illegible] AEB [illegible]

[illegible]

[illegible] Fig. 97. The Stress D[illegible] [illegible] the [illegible] [illegible] points [illegible] AEB, ABC, CBEGD, ACDE, EDGH.

It is important to notice that [illegible] CD Fig. 97 is tension, whereas [illegible] CD Fig. 98 is in compression, but the magnitude of [illegible] stress is, in both cases, the same. That is, **provided the load be kept constant in magnitude and position,** the [illegible] of the member CD merely [illegible] [illegible]

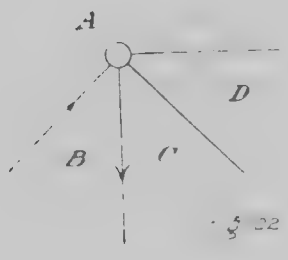


Fig. 22

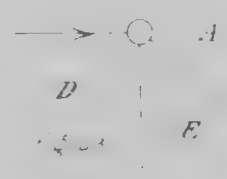


Fig. 23

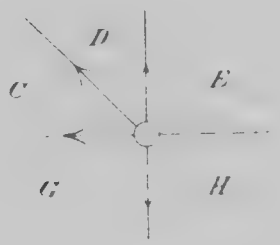


Fig. 24

[illegible]

The French Roof Truss.

[illegible]

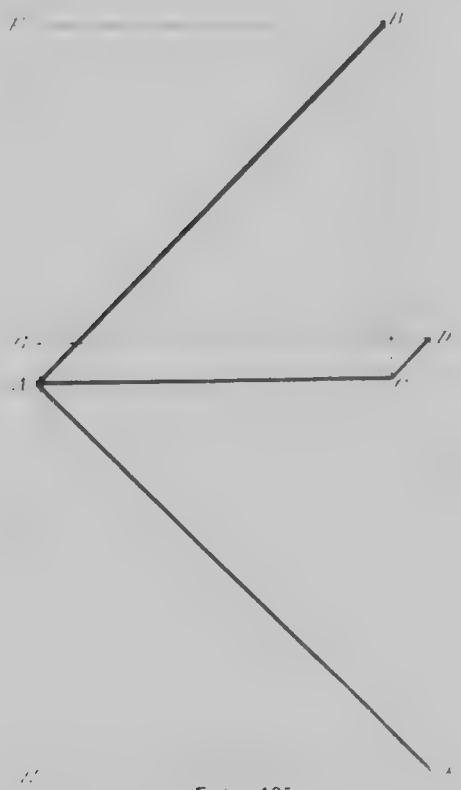


FIG. 105.

The Link Roof Truss.

[illegible]

Analytical Solution

Fig. 106.

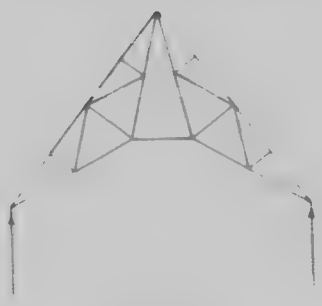


Fig. 107.

Effective Loads and Abutment Reactions.

In reality, there

(the effective loads)

the effective abutment reactions, since they are not the

determining the total

abutment reactions.

it will only be necessary to determine the stresses in the left

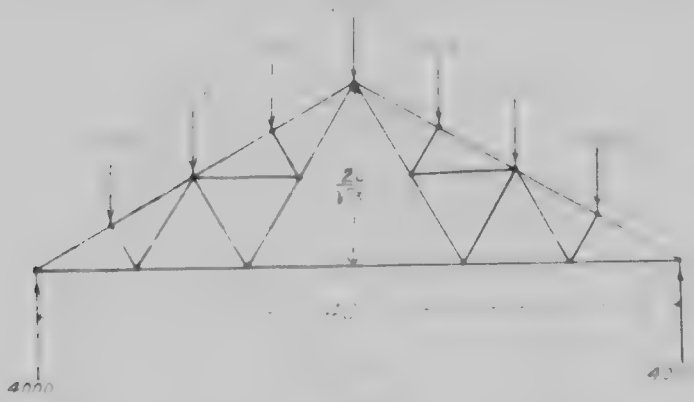


Fig. 108.

hand half of the truss, for, since [illegible] symmetrical, the stresses in corresponding members of both halves of the truss [illegible]

[illegible] position, it will be found that the [illegible]

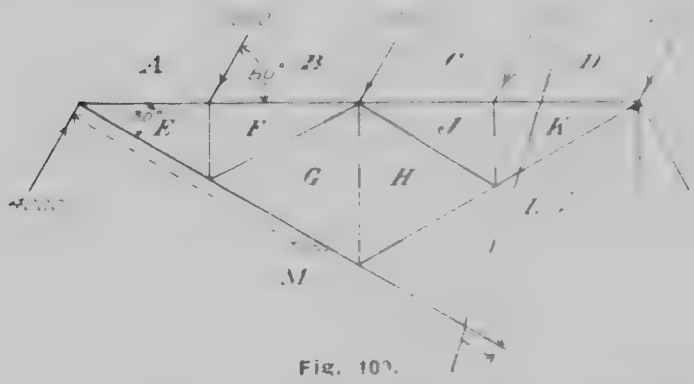


Fig. 109.

[illegible]

Consider the Point AME (Statical D

Consider the Point BAEF

Consider the Point EEMG [illegible]

[illegible]

[illegible]

[illegible] MG [illegible] GF [illegible]

[illegible]

MG [illegible] 4,000 x 3 [illegible] GF [illegible]

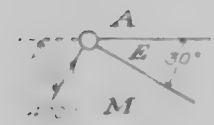


Fig 110

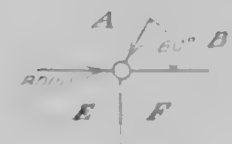
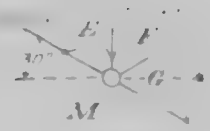


Fig [illegible]



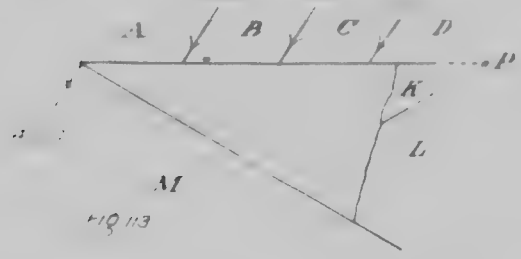


Fig 113

In [illegible] ordance with the X_{MG} having be[illegible] [illegible]ive, the Y_{MG} must be assumed negative if the [illegible] [illegible] to be used simultaneously. In the same [illegible] consideration of the line of action of GF, the Y must be assumed positive if the X [illegible]

$$-Y \quad Y_{EE} \quad Y_{EM} \quad Y_{[illegible]} \quad Y_{[illegible]}$$

[illegible] x 3 + 4,000 x 3, sin 30 [illegible] MG [illegible]

GF [illegible]

[illegible] x 3 + 4,000 x 3, [illegible] MG [illegible] GF [illegible]

Substitute value of MG from (1.) into [illegible]

[illegible] x 3 + 2,000 x 3 [illegible] (4,000 x 3 [illegible] GF [illegible] GF

[illegible] x 3 + 2,000 x 3 [illegible] 2,000 x 3 [illegible] GF [illegible] GF

GF [illegible] 500 x 3 [illegible]

[illegible] positive [illegible] GF [illegible] [illegible] to the X and Y of GF [illegible] [illegible] X and

Consider the Point GMLH

Graphical Solution

the point MAE.

[illegible] therefore, in Compression, and the member EM in Tension.

[illegible] discussion it will be merely necessary to [illegible] members [illegible] in tension [illegible] in finding the magnitudes of these [illegible] the figures can only be done by either [illegible] the lengths of the lines; which is impracticable, [illegible] polygons are not of a size suitable to any [illegible] by applying geometrical reasoning [illegible] figures, which may easily be done by the reader.

[illegible] of course, always best for the reader to [illegible] polygons or figures for himself, using some [illegible] scale.

Consider the point EABF. Fig. 116 is the Statical Diagram for the forces acting at this point. From the Vector Polygon, Fig. 117, it is seen that the forces BE

and EF both act against the point. The members BE and EF are, therefore, in Compression.

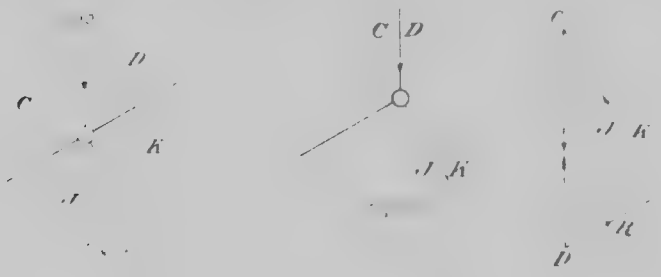
Considering the forces acting at the point MEFG (Statical Diagrams, Fig. 118). From the Vector Polygon, Fig. 119, it is seen that both the forces FG and GM [illegible] from the point, the members FG and GM being, in consequence, in Tension. (The reader will

[illegible] Vector Polygon [illegible] ME, EF, FG [illegible] GM [illegible] ME [illegible] GM below.

Consider, next, the point CDKJ (Statical Diagrams, [illegible] CD

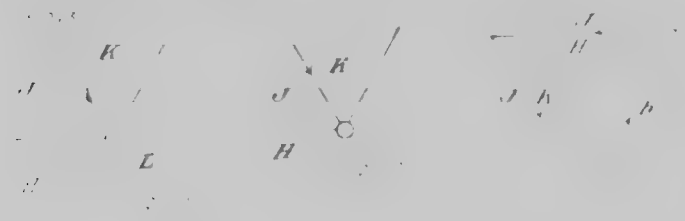
[illegible] and DK, KL and JC, [illegible] Since [illegible] three unknowns at this point, it is impossible [illegible] construct a Vector Polygon as the problem [illegible] JC and DK, [illegible] [illegible] [illegible] [illegible] [illegible]

Replace JC and DK by their resultant R [illegible] indicated in Statical Diagram, Fig. 121. Constructing the Vector Polygon [illegible] [illegible] it is seen from Fig. [illegible] [illegible] KL [illegible] against the point. The [illegible] K



is, therefore, in Compression. The magnitude of the resultant R gives no information of value.

Consider the point JKLH (Statical Diagram, Fig. [illegible]). Since KL and LH also have the same line of action, their resultant will act in that line. Replacing KL and LH by their resultant R, a condition such as shown in Statical Diagram, Fig. [illegible] [illegible] [illegible]



[illegible] Vector [illegible] [illegible] HL [illegible] [illegible] HL [illegible]

Coming back to the point GFBCJH [illegible]

[illegible]

[illegible] Vector Polygon [illegible]

[illegible] Vector Polygon [illegible] GL, FB, [illegible] BC [illegible] CL [illegible] BC [illegible] HL [illegible] Bow's Notation [illegible]

[illegible] HL [illegible]

[illegible] Vector Polygon [illegible] GL, FB, BC, HL [illegible] HG [illegible] Bow's Notation [illegible]

[illegible] Vector Polygon [illegible] Bow's Notation [illegible]

[illegible] HL [illegible] HL [illegible] HL

[illegible] Vector Polygon [illegible] GL, FB, BC, CL, HL, [illegible] HG [illegible] CL [illegible] HG [illegible] CL [illegible] HG [illegible] Vector Polygon [illegible]

to construct the polygon, using Bow's Notation, the Stress Diagram is to be drawn.

Stress Diagram.

Fig. 128 is the complete Stress Diagram for the left-hand half of the Fink Truss as shown in Fig. 128A. It will merely be necessary to point out the various Vector Polygons in this diagram since the construction of these polygons has been already gone through.

MA, AE, and EM (Fig. 128) evidently form a Vector Polygon for the forces acting at **the point MAE.** (See Figs. 114 and 115.)

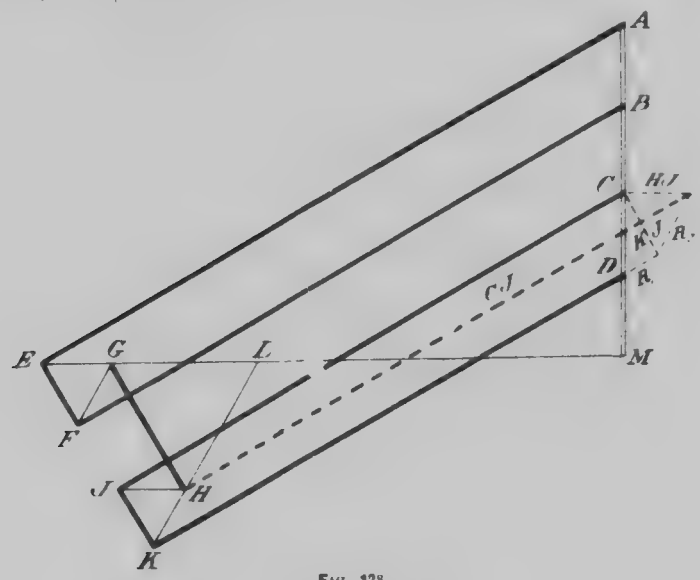


Fig. 128.

Considering **the point EABF,** the Vector Polygon reads on Fig. 128: EA, AB, BF, and FE. (Compare with Fig. 117.)

Coming next to **the point MEFG** (see Figs. 118 and 119), ME, EF, FG, and GM (Fig. 128) form a Vector Polygon for the forces acting at this point.

From B, on the line MA, lay off BC to represent the load BC. Then from C, on the same line, lay off CD to represent the load CD.

Consider **the point KJCD.** Replacing DK and JC by their resultant R_1, it is seen that CD, R (dotted), and KJ (dotted) form the Vector Polgon for the forces indicated in Statical Diagram, Fig. 121.

[illegible] **the point JKLH,** [illegible] KL and LH [illegible] R, which gives [illegible] Stat[illegible] Diagram [illegible] Fig. [illegible] JK dotted, R dotted, and HL dotted [illegible] the Vector Polygon [illegible] Compare with Fig. [illegible]

From a [illegible] **the point GFBCJH,** [illegible] GF, FB, BC, JH dotted [illegible] CJ dotted [illegible] and HG [illegible] Vector Polygon [illegible] Bow's Notation [illegible]

[illegible] Vector Polygon [illegible] Bow's Notation [illegible] applied [illegible] GF, FB, BC, CJ [illegible] JH [illegible] and HG [illegible]

[illegible] Stress Diagram [illegible] beyond

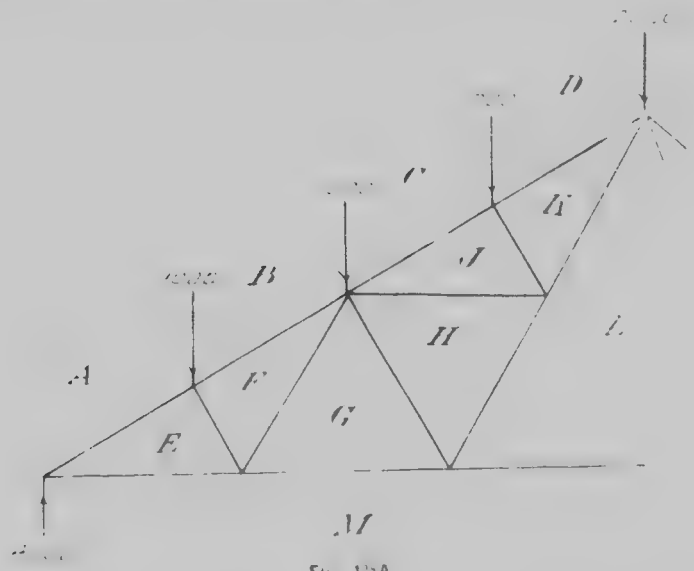
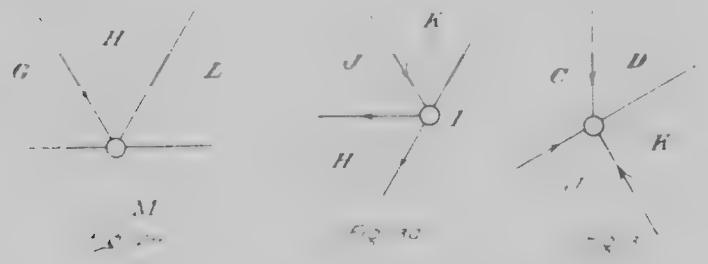


Fig. 15A

[illegible]

Referring to Statical Diagram (Fig. 129) for **the point MGHL**, it is seen that MG, GH, HL, and LM (Fig. 128) constitute a Vector Polygon for the forces acting at this point. It is evident from this polygon that both the forces HL and LM act away from the point. The members HL and LM are, therefore, both in Tension.



Consider **the point LHJK** (Statical Diagram, Fig. 130). JH, HL, JK, and KL (Fig. 128) is the Vector Polygon for the forces acting at this point, from which it is seen that the member KL is in Tension.

Coming back, finally, to **the point KJCD** (Statical Diagram, Fig. 131), evidently KJ, JC, CD, and DK (Fig. 128) form a Vector Polygon for the forces being considered. The force DK is seen to act against the point. The member DK must, in consequence, be in Compression.

The Pratt Truss.

Fig. 133 represents a form of bridge truss known as a Pratt Truss. The Effective Dead Load shown in the diagram has been chosen purely arbitrarily without regard to the practical rules which govern the first assumptions as to dead loads on bridges, convenient quantities having been chosen which will serve to illustrate the method of procedure.

Since the truss (Fig. 133) is symmetrically constructed and loaded, it will merely be necessary to consider one half of the structure, for corresponding members in both halves will have the same stresses.

The required stresses may be determined by either considering the forces acting on the pin at each joint of the truss, or the Method of Sections may be applied [illegible]

way as to give not more than three unknowns to each problem. The former course will be pursued in this case.

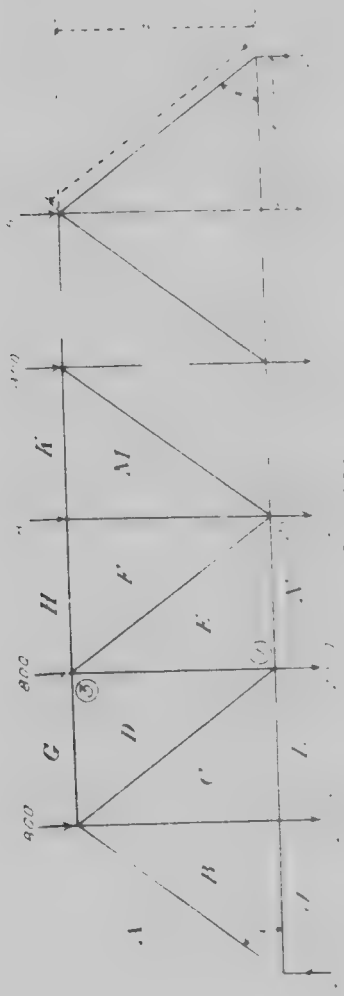


Fig. 133

Consider the forces acting on the pin at the joint BAJ (Statical Diagram, Fig. 134).

$$\Sigma Y = Y_{AJ} + Y_{JB} + Y_{BA} = 0.$$

$$7{,}000 + 0 + BA \cdot \tfrac{4}{5} = 0.$$

$$BA = \tfrac{5}{4} \cdot 7{,}000 = 8{,}750$$

The Y of BA is evidently negative. The force BA, therefore, acts against the point; i.e., the member BA is in Compression 8,750 pounds.

$$\Sigma X = X_{AJ} + X_{JB} + X_{BA} = 0$$
$$0 + JB - 8{,}750 \cdot 3/5 = 0$$
$$JB = 5{,}250$$

The positive result shows that X_{JB} is positive; i.e., JB acts to the right away from the point. The member JB is, therefore, in Tension 5,250 pounds.

Fig. 135 is the Statical Diagram for the forces acting at the point LJBC.

$$\Sigma X = X_{FL} + X_{JB} + X_{BC} - X_{CL} = 0$$
$$0 + -5{,}250 + 0 - CL = 0.$$
$$CL = 5{,}250$$

The member CL is evidently in Tension 5,250 pounds.

$$\Sigma Y = Y_{FL} - Y_{JB} - Y_{BC} - Y_{CL} = 0.$$
$$2{,}000 + 0 + BC - 0 = 0.$$
$$BC = 2{,}000.$$

The member BC is in Tension 2,000 pounds.

For the point CBAGD (Statical Diagram, Fig. 130)

$$\Sigma Y = Y_{CB} + Y_{BA} + Y_{AG} + Y_{GD} + Y_{DC} = 0$$
$$-2{,}000 + 8{,}750 \cdot 4/5 - 800 + 0 + DC \cdot 4/5 = 0$$
$$DC = -5{,}250$$

The member DC is in Tension 5,250 pounds.

$$\Sigma X = X_{CB} + X_{BA} + X_{AG} + X_{GD} + X_{DC} = 0.$$
$$0 + 8{,}750 \cdot 3/5 + 0 + GD + 5{,}250 \cdot 3/5 = 0.$$
$$GD = -8{,}400$$

The member GD is in Compression 8,400 pounds.

Considering the forces acting at the point NLCDE (Statical Diagram, Fig. 137):

$$\Sigma X = X_{NL} + X_{LC} + X_{CD} + X_{DE} + X_{EN} = 0$$

$$0 - 5{,}250 - 5{,}250 \cdot 3/5 + 0 + EN = 0$$

$$EN = 8{,}400$$

The member EN is in Tension 8,400 pounds.

$$\Sigma Y = Y_{NL} + Y_{LC} + Y_{CD} + Y_{DE} + Y_{EN} = 0$$

$$2{,}000 + 0 + 5{,}250 \cdot 4/5 + DE + 0 = 0$$

$$DE = -2{,}200$$

The member DE is in Compression 2,200 pounds.

Statical Diagram (Fig. 138) represents the condition at the point EDGHF.

$$\Sigma Y = Y_{FD} + Y_{DG} + Y_{GH} + Y_{HF} + Y_{FE} = 0$$

$$2{,}200 + 0 - 800 + 0 + FE \cdot 4/5 = 0$$

$$FE = -1{,}750$$

The member FE is in Tension 1,750 pounds.

$$\Sigma X = X_{FD} + X_{DG} + X_{GH} + X_{HF} + X_{FE} = 0$$

$$0 + 8{,}400 + 0 + HF + 1{,}750 \cdot 3/5 = 0$$

$$HF = -9{,}450$$

The member HF is in Compression 9,450 pounds.

From inspection of the point FHKM, it is easily seen that the member FM must be in Compression 800 pounds.

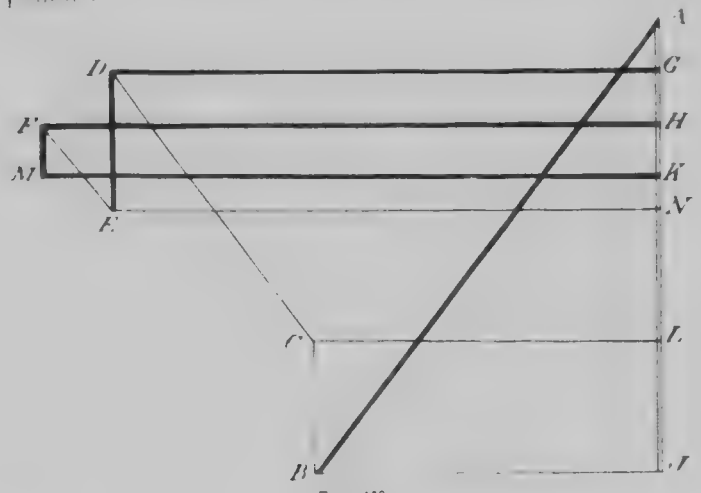


Fig. 139

Stress Diagram

The Stress Diagram for the Pratt Truss, loaded as indicated in Fig. 133, is shown in Fig. 139. If the sets

of forces acting at the various joints of the truss be considered in the same order as taken up in the preceding analytical solution (see Statical Diagrams, Figs. 134 . . . 138), the Vector Polygons for the respective cases may easily be followed out on the diagram.

Live Load Stresses.

A few structural details will have to be pointed out in order that the reader may clearly understand the following discussion of Live Load stresses:

Fig. 133 represents the truss on one side of a bridge. On the other side is a truss of the same construction. Connecting these trusses at the top is a system of bracing, and at the lower joints provision is made to

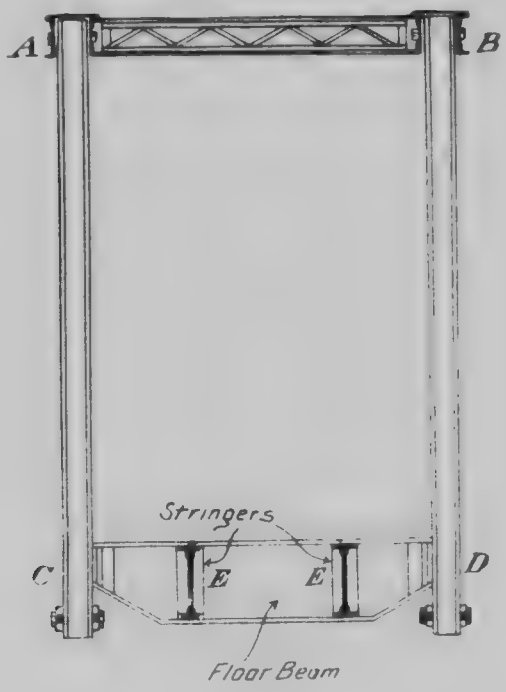


Fig. 14

support the floor system. Fig. 140 represents diagrammatically by an end elevation, the method of supporting the flooring, etc. This figure represents an end elevation of the joints (1.) and (3.) (Fig. 133) and the corresponding joints on the opposite truss of the bridge.

AC and BD represent the vertical members [illegible] the top between A and B. Riveted to the lower extremities of these vertical members is a beam, fastened to which are as many other longitudinal stringers. EE may be thought desirable to support the floor upon which the Live Load will pass. In this manner, the Live Load is transferred by the stringers EE to the floor beam placed between verticals at C and D, and from thence to the joints of the truss.

If now, a Live Load be in some position between the lower joints (1) and (2) (Fig. 133), it is seen from the preceding discussion that part of the load will be transferred to the joint (1) and part to the joint (2), the portion of the load transferred to either joint depending upon the distance of the load from (1) or (2).

It is seen from the preceding discussion on the construction of the floor beams and stringers that the live load is transferred by these beams, etc., to the joints of the truss. If a Live Load occupies a position between two joints, part of the load goes to one joint and part to the other joint, so that if the stress in any particular member is desired, **the truss must be considered as acted upon by the portions of the load transferred to the respective joints between which the load lies.** For instance, if the load were to lie half way between the joints (2) and (3) (Fig. 141), half of the load goes to (2) and half to (3). The truss should then be considered as acted upon by two forces, one at (2) and the other at (3), and, in this case, both equal to half the load.

It is merely necessary, for the time being, to consider the Live Load as it comes directly over each joint, in which case the total load is taken as acting on the particular joint at which it lies.

Fig. 141.

In Fig. 141 the dotted line represents the level along which the Live Load of 6,000 pounds passes.

Consider the load at joint (2). The left-hand abut-
[illegible] reaction when the load is in this position will be

[illegible] 5,000, since the load is [illegible] the total

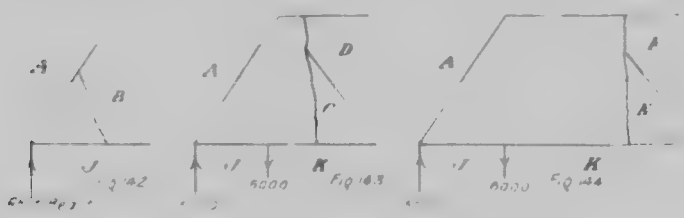
[illegible] from the left abutment.

Take a section through the members AB and BJ. Consider the forces acting on the portion of the truss to the left of this section. (This portion to the left may be considered as a solid body.) The forces acting on this body are in equilibrium, and act as represented in Fig. 142.

$$\Sigma Y = Y_{IA} - Y_{AB} - Y_{BJ} = 0$$
$$5{,}000 - 4.5\,AB = 0 = 0.$$
$$AB = 6{,}250.$$

AB is evidently in Compression 6,250 pounds.

Consider the forces acting on the portion of the truss to the left of a section through the members AD, DC, and CK. In this case, and all of the following cases, the area underneath the lower chords to the left of the load will be lettered J, and the area to the right of the load lettered K. It is seen that these areas will change whenever the load takes up a new position. (See Fig. 143.)



$$\Sigma Y = Y_{IA} - Y_{AD} - Y_{DC} - Y_{CK} - Y_{KJ} = 0$$
$$5{,}000 - 0 - DC \cdot 4.5 - 0 - 6{,}000 = 0$$
$$DC = 5 \cdot 4 \cdot 1{,}000.$$
$$= 1{,}250.$$

The member DC is in Compression 1,250 pounds.

By taking a section through the members AF, FE, and EK and considering the forces acting on the truss to the left of this section (see Fig. 144), the member FE may also be shown to be in Compression 1,250 pounds.

Consider the load at joint (3). The left-hand abut-
ment reaction in this case is $\frac{2}{3} \cdot 6{,}000 = 4{,}000$ pounds.

[illegible]

[illegible] EB, [illegible] [illegible]

[illegible]

AB [illegible]

AB [illegible] Compression [illegible]

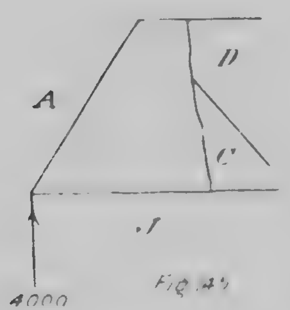
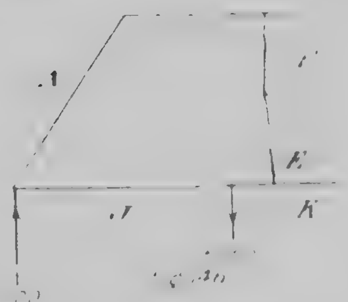


Fig. 45



[illegible]

Consider the forces acting on the section [illegible] to the left of a section through AD, DC, and CJ. See Fig. [illegible]

[illegible] DC [illegible]

5

DC [illegible]

4

5,000

[illegible] DC [illegible] Tension 5,000 pounds.

Fig. [illegible] represent the forces acting on the section [illegible] to the left of section through AE, EK, [illegible]

[illegible] EK [illegible] 2,000 [illegible]

5

EK [illegible] 2,500

4

Hence the bar EK is in Compression 2,500 pounds.

Consider the load at joint (4). In this case, [illegible] Live Load will be supported by each abutment; [illegible]

[illegible] AB [illegible] Method of [illegible] to be in Compression 3,750 pounds, and the members DC and FE both in Tension 3,750 pounds.

[illegible]

The Dead and Live Load Stresses having been determined independently of one another, the combined effect of Dead and Live Load may now be examined.

As previously pointed out, the Dead Load is composed of the weight of the truss and any fixed objects that may be on the bridge. This being the case, the stresses determined by considering the Dead Load alone will exist when the bridge is not in use. Furthermore, because the Dead Load is always on the truss, there will always exist the tendency to produce these Dead Load Stresses, although, as will be seen, these stresses may be modified, and in some cases reversed, from Tension to Compression by the Live Load.

A plus sign before a value in Fig. 147 indicates Tension, a negative sign indicating Compression.

Considering first the member AB, it is seen from Fig. 147 that this member is always in Compression. The maximum combined stress due to Dead and Live Load is 15,000 pounds, given when the Live Load is over joint (2) (Fig. 141).

Member	Dead Load Stresses	Live Load at 2	Live Load at 3	Live Load at 4	Maximum Stresses
AB	-8750	-6250	-5000	-3750	-15000
DC	+5250	-1250	+5000	+3750	+10250
FE	+1750	-1250	-2500	+3750	+5500

Fig. 147.

Consider the member DC. This member is in Tension 5,250 pounds, due to the Dead Load. The effect of the Live Load taking up a position at joint (2) is to destroy 1,250 pounds of the Dead Load Tension.

The Live Load **alone** at joints (3) and (4) places DC in Tension; that is, in either of these positions the Live Load augments the existing Tension due to Dead Load. It is evident that the maximum stress in DC is 10,250 pounds Tension, given when the Live Load is at joint [illegible]

Consider the member FE. The Dead Load place this member in Tension 1,750 pounds. The Live Load it moves to joint (2) is seen to destroy 1,250 pounds this Tension. Reaching joint (3), the Live Load not only destroys the existing Dead Load Tension, but is seen to place FE in Compression 750 pounds. **However, a Tension member due to its slender proportions cannot take up Compression without "buckling."** This being the case, provision must be made for this 750 pounds Compression, which would otherwise "buckle" FE.

The Counterbrace.

Let Fig. 148 represent the panel in which the member FE lies. In this diagram, the line joining A and B represents the member FE.

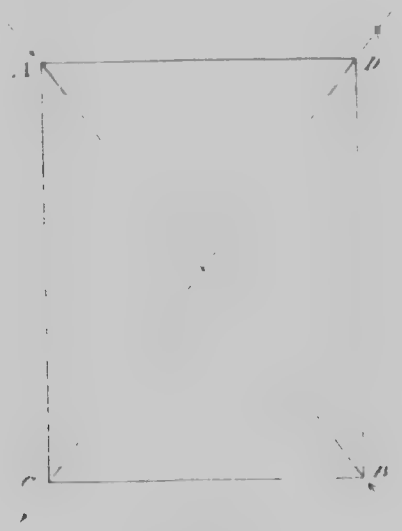


Fig. 148.

It has just been shown that when the Live Load reaches joint (3) (Fig. 141), the member FE would be subjected to 750 pounds Compression. This being the case, the condition of affairs may be represented by a pair of equal forces, one acting at each extremity of FE at A and B as indicated in Fig. 148, these two forces tending to place FE in Compression

Now, since FE cannot withstand Compression, the joints A and B will tend to move inward under the forces indicated. But when the joints A and B move inward,

the joints D and C will move outward. A Tension member placed from D to C, as indicated by the dotted line, will prevent D and C from moving outward, and, in consequence, prevent A and B from moving inward. That is, a Tension member placed from D to C capable of taking 750 pounds stress will relieve the member FE from any liability to being placed in Compression when the Live Load is at joint (3).

The member FE must be capable of taking 5,500 Tension, which is the maximum combined Dead and Live Load Stress. This stress occurs when the Live Load is at joint (4).

The above reasoning on the counterbrace, while of an almost self-evident nature, must be recognized by the reader as merely a popular explanation, which could hardly stand a scientific examination without further explanation.

The reader can easily see that it would be impractical to place a small compression member alongside FE to take up the excess Compression when the Live Load is at joint (3). If, however, a diagonal member be placed from the remaining corners of the panel, it is evident that **if FE be considered as not acting, and, therefore, left out of the problem**, that this new diagonal member will be in Tension when the Live Load arrives at joint (3). To prove this statement, **let the reader take out the member FE in Figs. 133 and 141** and replace it with a diagonal member placed from the other corners of the panel. If the Dead and Live Load stresses be determined for this new member and then combined, the member will be found in Tension 750 pounds.

Influence Lines.

A curve which shows the variation of stress in any particular member of a truss as the Live Load moves over the structure is known as an Influence Line.

The above definition is complete in as far as it goes, but it may be pointed out here that curves which show the variation in Shear and Bending Moment at any particular point or section of a body for a moving load are also known as Influence Lines.

Curves which show the variation of Stress, Shear or Bending Moment for different sections of a body while the load stays in one particular position are not called influence lines, but have names applied to them according to the function they represent.

APPENDIX.

This portion of the book, presented as an appendix, gives an extremely [illegible] method of solving problems which necessitate the use of the Funicular Polygon. The reader must not confuse the polygon derived by this method with the [illegible] of the Equilibrium Polygon, although, [illegible] geometrical outline [illegible], both are identical. In the following discussion the original load [illegible] is considered as indefinitely extended, and the original forces [illegible] which by their position counteract one another, thereby allowing the reader to reduce the forces to an [illegible] pair. The polygon so derived is, for want of a better name, called by the writer a Funicular Polygon, although it is freely recognized that, so applied, the name is not absolutely [illegible].

THE FUNICULAR POLYGON SECOND GRAPHICAL CONDITION OF EQUILIBRIUM.

Graphical Location of the Resultant of a Set of Forces.

It has been already shown that the resultant of a set of forces which act at a point also acts at the same point, thus making the problem of locating the resultant a very simple one for that particular case. There arise, however, numerous cases where the forces comprising a set do not act at a point, so that the previous method of locating the resultant does not apply. If the lines of action of the forces comprising the set intersect within reasonable limits, the resultant might be located as follows.

Let P, Q, S, and T [illegible] forces. [illegible] P and Q [illegible] R_1. R_1, S, and T [illegible] R_1 [illegible] S [illegible] R_2 [illegible] P, Q, S, and T [illegible] R_3 [illegible]

[illegible]

[illegible]

Let P, Q, and S [illegible] ABCD [illegible] AD.

[illegible] point O inside or outside of the vector polygon and join it to all points [illegible] represent[illegible] of the various forces, in this case, to A, B, C, and D. It might have been a little clearer, in this particular case, to [illegible] O to all corners of the vector polygon, but the above phraseology is more general and applies to the vector polygon of [illegible] possible forces.

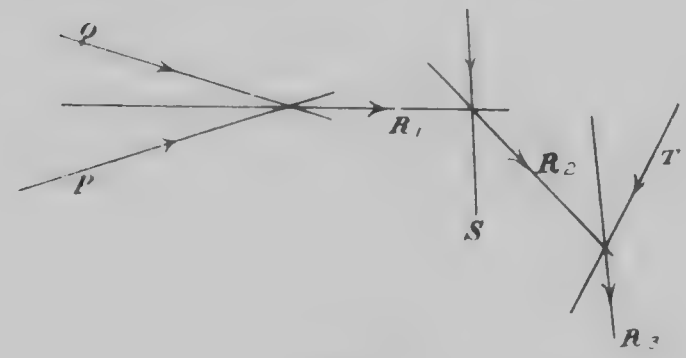


Fig. 30.

Before proceeding with the argument, there are two facts that must be borne in [illegible] throughout this proposition:

(1) **A force may be replaced by its components at any point on its line of action.**

(2) **The resultant of two forces acts through the intersection of the lines of action of the two forces.**

Referring back to Fig. 31, AO and OB represent a pair of components of the force represented by AB, i.e., the force P. P may, therefore, be replaced by AO and OB at any point on its line of action. See (1).] At any point X on the line of action of P introduce AO and OB. Produce the line of action of OB to intersect the line of action of Q at Y. Q may be replaced by the two components represented by BO and OC, and, since [illegible] be replaced at any point on its line of action, [illegible] the point Y. At Y introduce the components BO and OC. [illegible] BO [illegible]

Fig. 31.

The forces P, Q, and S have been replaced by their respective components, so that in place of the original forces there are now acting the forces AO, OB, BO, OC, CO, and OD. It is seen, however, that OB and BO are two forces of equal magnitude, acting with opposite senses in the same line of action. Their effect on the equilibrium of the body is, therefore, zero. In the same way, OC and CO counteract one another, thus leaving AO and OD, so that as far as actual effect is concerned **AO and OD are equivalent to the original forces.** But AO and OD are two components of AD, or conversely, AD is the resultant of the two forces, AO and OD; i.e., from (2), AO and OD may be replaced at the intersection of their lines of action (the point Z) by AD. AD, however, is the resultant of the original set of forces which it was required to locate.

The Funicular Polygon.

In Fig. 31 the polygon XYWZ, formed by the lines representing the directions of the replacing components is given the name of a **Funicular Polygon.** This polygon must not be confused in any way with the vector polygon, nor with the lines drawn from the point O to the vector polygon. This last mentioned point O is known as a "pole," which, together with the vector diagram and the lines to O, form a **Polar Diagram.**

The Resultant Couple.

There arise many cases of sets of forces where, if the vector polygon be drawn, it is found to close, and yet the forces are not in equilibrium. For instance, if in Fig. 32 the vector polygon ACB be drawn for the set of forces P, Q, and S as represented, it is found to close. But it is evident that such a set of forces would produce a rotation of the body; that is, there is not equilibrium, although the resultant force is zero.*

It will thus be seen that cases may arise when a set of forces acts upon a body, such that, although what we term the **resultant force** vanishes, there is yet a motion of rotation about some point in the body; and we speak of the forces as equivalent then to a **resultant couple.**

* Three forces to be in equilibrium must act at a point.

Draw the funicular polygon for P, Q, and S, and it will be found that the first and last components do not intersect one another, if their directions are produced, but that they have parallel lines of action.

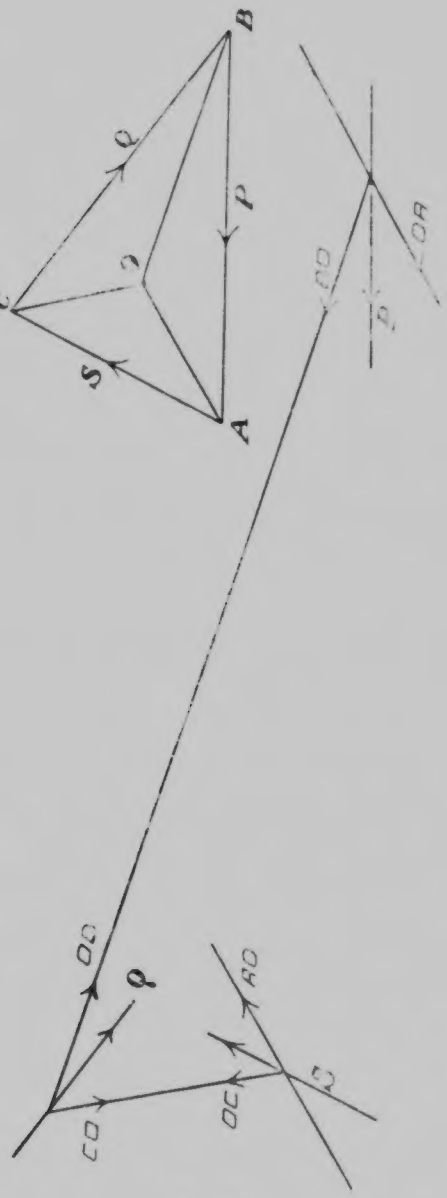


Fig. 32.

Since these first and last components are of equal magnitude and act in the opposite sense, they form a couple, which is called the **Resultant Couple.**

Second Graphical Condition of Equilibrium.

If in the last proposition and in the one preceding it, the lines of action of the first and last components had been coincident, these two components would then counteract one another, so that the effect of all the replacing components would be zero, or, in other words, the original forces would be in equilibrium.

So, then, as a second graphical condition of equilibrium, **the first and last components of the funicular polygon must have their lines of action coincident.**

This is often more concisely stated by saying that **the funicular polygon must close,** keeping in mind what is meant by the word "close."

Graphical and Analytical Determination of the Resultant.

Graphical.	Analytical.
Vector Polygon gives the magnitude, direction, and sense of the resultant.	$\Sigma X = X_R$. $\Sigma Y = Y_R$. $R = \sqrt{\Sigma X^2 + \Sigma Y^2}$. $\tan \alpha = \frac{\Sigma Y}{\Sigma X}$
Funicular Polygon locates the line of action of the resultant.	$\Sigma M = M_R$.

Conditions of Equilibrium.

Graphical.	Analytical.
Vector Polygon must close.	$\Sigma X = 0$. $\Sigma Y = 0$.
Funicular Polygon must close.	$\Sigma M = 0$.